

SECONDARY FOUR

PRELIMINARY EXAMINATIONS 2025



ADDITIONAL MATHEMATICS

4049/01

Paper 1 Marking Scheme

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiners' Use

Question	Marks	Question	Marks		
1		8			
2		9		Units	
3		10		Clarity / Logic	
4		11		Precision / Accuracy	
5		12		Total:	
6		13		90	
7		14			
Date:					

NOTE: *If the marks allocation is italicised, it has been modified from version 2 according to the suggestions given by the tester.*

- 1 (a) Find the range of values of p for which $y = x^2 - x - 2$ lies entirely above the line $y = p(x + 2)$. [4]

$x^2 - x - 2 = px + 2p$ $x^2 + (-1 - p)x - 2 - 2p = 0$ $\Rightarrow \text{discriminant} < 0$ $\Rightarrow (-1 - p)^2 - 4(1)(-2 - 2p) < 0$ $\Rightarrow 1 + 2p + p^2 + 8 + 8p < 0$ $\Rightarrow p^2 + 10p + 9 < 0$ $\Rightarrow (p + 9)(p + 1) < 0$ $\Rightarrow -9 < p < -1$	M1 M1 for discriminant < 0 M1 A1
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- (b) Hence, deduce, without finding the discriminant, the number of intersection points when $p = 3$. [1]

Line $y = p(x + 2)$ does not intersect curve $y = (x + 1)(x - 2)$ when p is in the range $-9 < p < -1$. For $p = 3$, line intersects curve at 2 points.	B1
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- 2 (a) By using substitution or otherwise, find the values of x for which $2^{2x-1} = 2^{x+3} - 24$, giving your answer where appropriate, to one decimal place. [5]

$2^{2x-1} = 2^{x+3} - 24$ $\frac{1}{2}(2^x)^2 = 2^3 \times 2^x - 24$ Let $u = 2^x$ $\frac{1}{2}(u)^2 = 2^3 \times u - 24$ $\frac{1}{2}u^2 = 8u - 24$ $u^2 = 16u - 48$ $u^2 - 16u + 48 = 0$ $(u - 12)(u - 4) = 0$ $u = 12 \quad \text{or} \quad u = 4$ $2^x = 12 \quad \text{or} \quad 2^x = 4$ $\lg 2^x = \lg 12 \quad \text{or} \quad 2^x = 2^2$ $x = \frac{\lg 12}{\lg 2} \quad \text{or} \quad x = 2$ $x \approx 3.58496$ ≈ 3.6	M1 applying Laws of Indices to break them into product of two numbers M1 for getting quadratic equation M1 for using lg to solve. A2 for both answers
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- 3 A closed circular cylinder has a volume of $(66\sqrt{2} + 2\sqrt{3})\pi \text{ cm}^3$, a radius of $(\sqrt{2} + 2\sqrt{3}) \text{ cm}$ and a height $h \text{ cm}$. Express h in the form of $p\sqrt{2} + q\sqrt{3}$ where p and q are integers. [4]

Volume of cylinder = $(66\sqrt{2} + 2\sqrt{3})\pi$ $\pi r^2 h = (66\sqrt{2} + 2\sqrt{3})\pi$ $(\sqrt{2} + 2\sqrt{3})^2 h = (66\sqrt{2} + 2\sqrt{3})$ $(2 + 4\sqrt{6} + 12)h = (66\sqrt{2} + 2\sqrt{3})$ $h = \frac{66\sqrt{2} + 2\sqrt{3}}{14 + 4\sqrt{6}}$ $h = \frac{66\sqrt{2} + 2\sqrt{3}}{14 + 4\sqrt{6}} \times \frac{14 - 4\sqrt{6}}{14 - 4\sqrt{6}}$ $h = \frac{924\sqrt{2} - 264\sqrt{12} + 28\sqrt{3} - 8\sqrt{18}}{196 - 96}$ $h = \frac{924\sqrt{2} - 528\sqrt{3} + 28\sqrt{3} - 24\sqrt{2}}{100}$ $h = \frac{900\sqrt{2} - 500\sqrt{3}}{100}$ $h = 9\sqrt{2} - 5\sqrt{3}$	M1 – expansion of r^2 M1 – rationalisation M1 – multiplication and simplification A1 for answer
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- 4 Express $\frac{2x^3 - 2x^2 + 7x - 12}{x^3 + 4x}$ in partial fractions. [6]

2 $\begin{array}{r} x^3 + 4x \overline{) 2x^3 - 2x^2 + 7x - 12} \\ \underline{-(2x^3 + 8x)} \\ -2x^2 - x - 12 \end{array}$ $\therefore \frac{2x^3 - 2x^2 + 7x - 12}{x^3 + 4x} = 2 + \frac{-2x^2 - x - 12}{x^3 + 4x}$ $\frac{-2x^2 - x - 12}{x^3 + 4x} = \frac{A}{x} + \frac{Bx + C}{x^2 + 4}$ Multiply by $(x^3 + 4x)$ throughout, $-2x^2 - x - 12 = A(x^2 + 4) + x(Bx + C)$ Sub $x = 0$, $-12 = A(4) + 0$ $A = -3$	Marking Scheme modified from Version 2 B1 for long division working M1 for breaking into partial fractions A1 for value of A
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By comparing coefficients of , $A + B = -2$ $-3 + B = -2$ $B = 1$ $\frac{2x^3 - 2x^2 + 7x - 12}{x^3 + 4x} = 2 + \frac{x-1}{x^2+4} - \frac{3}{x}$	$A1$ for value of B and of C . $A1$ for final answer
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5 Given the curve $y = kx^2 + 2kx - 3$, where k is a constant,

(i) By expressing y in the form $k(x+b)^2 - 1$, determine the value of k and b . [3]

(i)	$y = kx^2 + 2kx - 3$ $y = k(x^2 + 2x) - 3$ $y = k(x^2 + 2x + 1 - 1) - 3$ $y = k(x+1)^2 - k - 3$ $b = 1$ $-k - 3 = -1$ $k = -2$	Marking Scheme modified from Version 2 $M1$ for $y = k(x+1)^2 - k - 3$ $A1$ for value of b $A1$ for value of k
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(ii) Hence, show that the curve is always below the x -axis. [2]

(ii)	$y = -2(x+1)^2 - 1$ $(x+1)^2 \geq 0$ $-2(x+1)^2 \leq 0$ $-2(x+1)^2 - 1 \leq -1 < 0$ Hence, the graph of y is always below the x -axis.	$M1$ $A1$
	or	
	Discriminant $= (-4)^2 - 4(-2)(-3) = -8 < 0$ Since discriminant < 0 , the curve does not intersect the x -axis. Coefficient of x^2 is also negative, hence it is a quadratic curve with a maximum point. Hence, the curve is always above the x -axis.	$M1$ discriminant $A1$ for negative coefficient of x^2
	or	
	The turning point is $(-1, -1)$ and is a maximum point. Since the y -coordinate is less than zero, the curve is always above the x -axis.	$M1$ for maximum turning point. $A1$ for $y\text{-coor} < 0$

(iii) State the turning point of the curve and determine the nature of this point. [2]

(iii)	turning point = $(-1, -1)$ and is a maximum point	B1 B1
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6 The binomial expansion of $(1 + px)^n$ where $n > 0$, in ascending powers of x is

$$1 - 12x + 28p^2x^2 + qx^3 + \dots$$

Find the value of p , of n and of q .

[6]

$(1 + px)^n$ $= 1 + \binom{n}{1}(px) + \binom{n}{2}(px)^2 + \binom{n}{3}(px)^3 + \dots$ $= 1 + np x + \frac{n(n-1)}{2} p^2 x^2 + \frac{n(n-1)(n-2)}{6} p^3 x^3 + \dots$ $= 1 - 12x + 28p^2x^2 + qx^3 + \dots$ By comparing coefficients of x^2, $\frac{n(n-1)}{2} = 28$ $n^2 - n = 56$ $n^2 - n - 56 = 0$ $(n-8)(n+7) = 0$ $n = 8 \quad \text{or} \quad n = -7 \text{ (N.A.)}$ By comparing coefficients of x, $np = -12$ $p = \frac{-12}{8}$ $p = -1\frac{1}{2}$ $q = \frac{8(7)(6)}{6} \left(-1\frac{1}{2}\right)^3$ $q = -189$	Marking Scheme modified from Version 2 <i>B1 for applying the formula for Binomial Coefficient</i> <i>M1 for forming equation. $\binom{n}{2}p^2 = 28$ is not acceptable</i> <i>A1 for both answers, must reject -7.</i> <i>M1 for forming equation. $\binom{n}{3}p = -12$ is not acceptable</i> <i>A1 for value of p</i> <i>A1 for value of q</i>
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- 7 (a) The surface area of a solid cube is increasing at $0.2 \text{ cm}^2/\text{s}$. Find the rate of increase of the volume when the length of a side is 1 cm.

[4]

$A = 6x^2$ $\frac{dA}{dx} = 12x$ $\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}$ $0.2 = (12x) \frac{dx}{dt}$ $\frac{dx}{dt} = \frac{1}{60x}$ $V = x^3$ $\frac{dV}{dx} = 3x^2$ $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$ $\frac{dV}{dt} = 3x^2 \left(\frac{1}{60x} \right)$ $\frac{dV}{dt} = \frac{x}{20}$ Subst. $x = 1$, $\frac{dV}{dt} = \frac{1}{20}$ $\frac{dV}{dt} = 0.05 \text{ cm}^3/\text{s}$	M1 – form correct connect rates of change M1 – find $\frac{dx}{dt}$ M1 – form correct connect rates of change A1

9 (a) Solve the equation $\log_2(x+2) - \log_{\sqrt{2}}(x-1) = 1$.

[4]

$\log_2(x+2) - \log_{\sqrt{2}}(x-1) = 1$ $\log_2(x+2) - \frac{\log_2(x-1)}{\log_2 \sqrt{2}} = 1$ $\log_2(x+2) - \frac{\log_2(x-1)}{\frac{1}{2}} = 1$ $\log_2(x+2) - \frac{\log_2(x-1)}{\frac{1}{2} \log_2 2} = 1$ $\log_2(x+2) - \frac{\log_2(x-1)}{\frac{1}{2}} = 1$ $\frac{1}{2} \log_2(x+2) - \log_2(x-1) = \frac{1}{2}$ $\log_2(x+2) - 2 \log_2(x-1) = 1$ $\log_2\left(\frac{x+2}{(x-1)^2}\right) = 1$ $\frac{x+2}{(x-1)^2} = 2$ $x+2 = 2(x^2 - 2x + 1)$ $x+2 = 2x^2 - 4x + 2$ $2x^2 - 4x + 2 - x - 2 = 0$ $2x^2 - 5x = 0$ $x(2x - 5) = 0$ $x = 0 \quad \text{or} \quad 2x - 5 = 0$ $x = 0 \text{ (N.A.)} \quad \text{or} \quad x = 2\frac{1}{2}$	Marking Scheme modified from Version 2 M1 for using Change-of-base Law M1 for using Quotient Law <i>M1 for removing log either by changing to index form or changing 1 to $\log_3 3$ and remove the $\log_3$</i> A1 for both answers. Must reject 0.
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(b) It is given that $\lg a - \lg b = \lg(a + b)$.

(i) Express a in terms of b .

[2]

$\lg a - \lg b = \lg(a + b)$ $\lg \frac{a}{b} = \lg(a + b)$ $\frac{a}{b} = (a + b)$ $a = ab + b^2$ $a - ab = b^2$ $a = \frac{b^2}{1 - b}$	M1 for using Quotient Law A1 for answer
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(ii) State the range of b .

[1]

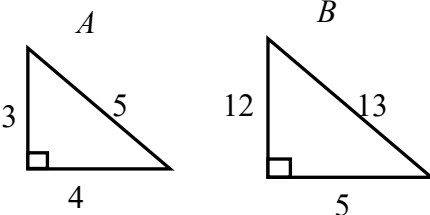
Since $a > 0$, $\therefore \frac{b^2}{1 - b} > 0$ $\therefore b^2 > 0$, $\therefore 1 - b > 0$ $b < 1$ $\therefore 0 < b < 1$	B1
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10 Given that $\sin A$ and $\cos B$ are in the same quadrant such that $\sin A = \frac{3}{5}$ and $\cos B = -\frac{5}{13}$.

Find, without using a calculator, the value of

(i) $\cos(A - B)$,

[2]

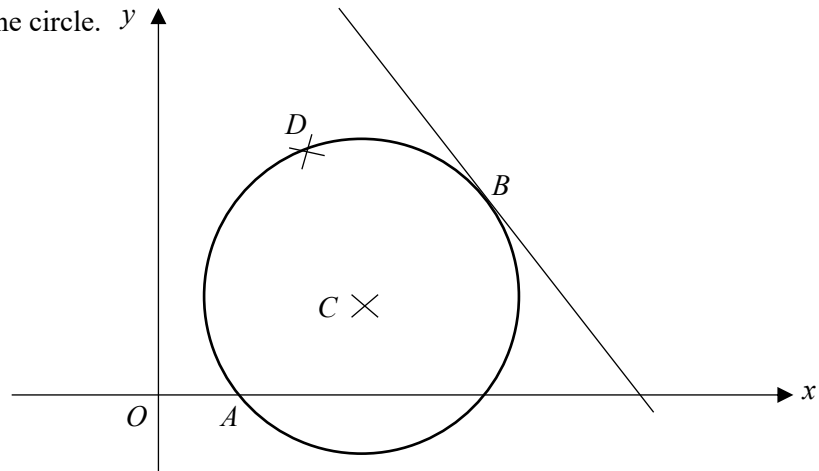
$\cos(A - B) = \cos A \cos B + \sin A \sin B$ $= \left(-\frac{4}{5}\right)\left(-\frac{5}{13}\right) + \left(\frac{3}{5}\right)\left(\frac{12}{13}\right)$ $= \frac{56}{65}$		M1 A1
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(ii) $\cos \frac{A}{2}$.

[3]

$\cos A = 2 \cos^2 \frac{A}{2} - 1$ $\cos A + 1 = 2 \cos^2 \frac{A}{2}$ $-\frac{4}{5} + 1 = 2 \cos^2 \frac{A}{2}$ $\frac{1}{5} = 2 \cos^2 \frac{A}{2}$ $\cos^2 \frac{A}{2} = \frac{1}{10}$ $\cos \frac{A}{2} = \pm \sqrt{\frac{1}{10}}$ $90^\circ < A < 180^\circ$ $45^\circ < \frac{A}{2} < 90^\circ$ $\cos \frac{A}{2} = \frac{\sqrt{10}}{10} \quad \left(-\frac{\sqrt{10}}{10} \text{ (NA)} \right)$	M1 M1 A1 with rejection
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- 11 Points A , $B(5, 4)$ and D lie on a circle with centre C and AB is the diameter of the circle. The line $y = -x + 9$ is a tangent at point $B(5, 4)$ on the circle. y $\uparrow$



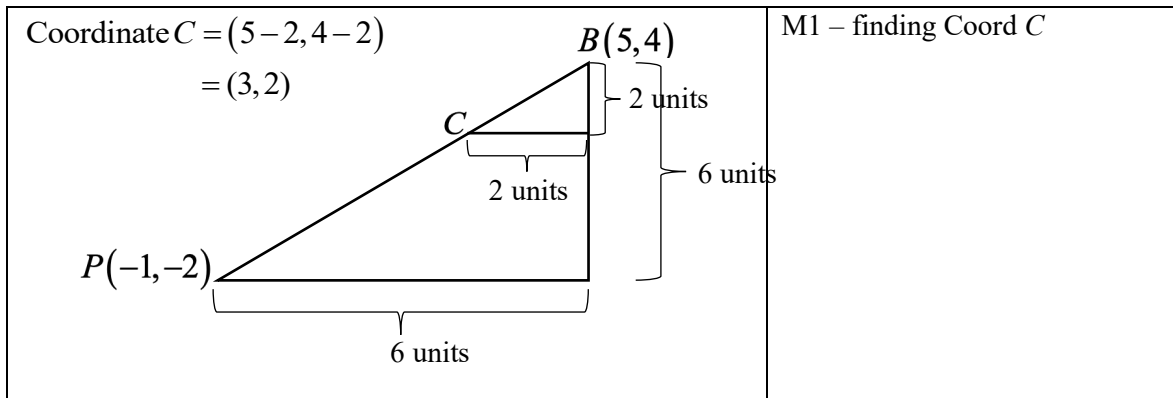
- (a) Given that point P lies on BA extended and the x -coordinate of point P is -1 , find the coordinates of P . [2]

Let the y-coordinate of P be a, $\frac{4-a}{5-(-1)} = 1$ $4-a = 6$ $a = -2$ Coordinates of P is $(-1, -2)$	M1 – form equation A1
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It is given that the distance BP is three times the distance BC .

(b) (i) Show that centre C is $(3, 2)$.

[1]



(ii) Hence, find the equation of the circle.

[2]

Distance $BC = \sqrt{(5-3)^2 + (4-2)^2}$ $= \sqrt{8}$ units Equation of circle is $(x-3)^2 + (y-2)^2 = 8$	M1 – finding distance BC A1
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(c) Given that triangle BCD is right-angled at C , find the coordinates of D .

[3]

Gradient of $CD = -1$ Equation of CD is $y - 2 = -1(x - 3)$ $y = -x + 5$ Subst. $y = -x + 5$ into $(x-3)^2 + (y-2)^2 = 8$, $(x-3)^2 + (-x+5-2)^2 = 8$ $x^2 - 6x + 9 + x^2 - 6x + 9 = 8$ $2x^2 - 12x + 10 = 0$ $x^2 - 6x + 5 = 0$ $(x-1)(x-5) = 0$ $x = 1$ or $x = 5$ (NA) Subst. $x = 1$ into $y = -x + 5$, $y = -1 + 5$ $y = 4$ $D(1, 4)$	M1 – find equation CD M1 – solving simultaneous equations A1
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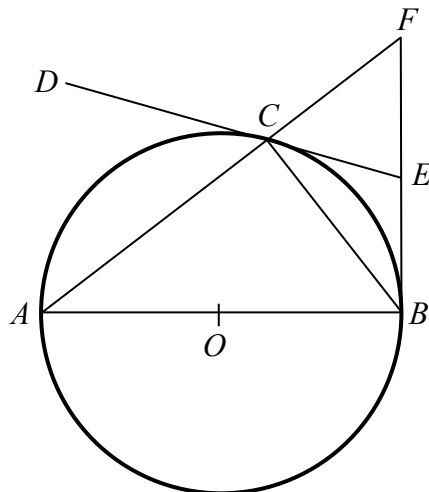
12 (a) Given that $y = \ln(\sin^2 x)$, show that $\frac{dy}{dx} = 2 \cot x$. [2]

$y = \ln(\sin^2 x)$ $= 2 \ln(\sin x)$ $\frac{dy}{dx} = 2 \left(\frac{1}{\sin x} \right) (\cos x)$ $= 2 \cot x$	M1: differentiation of $\ln$ M1: differentiation of $\sin x$
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(b) Hence show that the value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\cot x - \cos^2 2x) dx = a + b \ln 2$, where a and b are constants. [4]

$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x - \cos^2 2x dx = \frac{1}{2} \left[2 \ln \sin x \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}} - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos 4x + 1}{2} dx$ $= \frac{1}{2} \left[0 - 2 \ln \frac{1}{\sqrt{2}} \right] - \frac{1}{2} \left[\frac{\sin 4x}{4} + x \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ $= \frac{1}{2} \ln 2 - \frac{1}{2} \left[\frac{\pi}{2} - \frac{\pi}{4} \right]$ $= \frac{1}{2} \ln 2 - \frac{\pi}{8}$ $= -\frac{\pi}{8} + \frac{1}{2} \ln 2$	M1: for using (a) M1: for correct double angle formula M1: definite integrals A1
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- 13 In the diagram, AB is a diameter of the circle with centre O . DE and BF are tangents to the circle at C and B respectively. DCE and BEF are straight lines. Prove that



(a) $\triangle ABC$ and $\triangle BFC$ are similar,

[3]

$\angle ACB = 90^\circ$ (rt. $\angle$ in a semi-circle)	M1
$\angle FCB = 90^\circ$ (adj $\angle$ on straight line)	M1
$\therefore \angle ACB = \angle BCF$	
$\angle CBF = \angle CAB$ (tangent-chord theorem)	M1
Hence $\triangle ABC$ and $\triangle BFC$ are similar.	

(b) Hence or otherwise, show that $\triangle CFE$ is an isosceles triangle.

[4]

$\angle ABC = \angle DCA$ (tangent-chord theorem)	M1
$\angle FCE = \angle DCA$ (vertically opp. $\angle$ s)	M1
$\angle ABC = \angle CFE$ (similar Δ s)	M1
Since $\angle CFE = \angle FCE$, $\triangle CFE$ is isosceles (base $\angle$ s are equal)	A1

- 14 The mass, m grams, of the decomposition of a radio-active substance is given by the formula $m = ae^{-bt}$, where a and b are constants, and t is the number of weeks after the initial measurement of the mass. Experimental values of t and m are given in the table below.

t	1	2	3	4	5
m	1.51	0.454	0.137	0.0224	0.0124

- (i) What does the constant a refer to?[1]

The constant a refers to the initial mass of the radio-active substance. Or The constant a refers to the mass of the radio-active substance when $t = 0$.	B1
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- (ii) By drawing a suitable straight line graph using the data provided, estimate the value of a and of b . [7]

$m = ae^{-bt}$ $\ln m = \ln(ae^{-bt})$ $\ln m = \ln a - bt$ $\ln m = -bt + \ln a$						M1
						A1
						T1
t	1	2	3	4	5	
$\ln m$	0.412	-0.790	-1.99	-4.00	-4.39	

$\ln m = -bt + \ln a$

G1: Plot all the points

G2: Best fit line

Penalise under Clarity if missing in labelling of axis / equation

$$-b = \frac{-4 - 1}{4.7 - 0.5}$$

$$-b \approx -1.1904$$

$$b \approx 1.19$$

$$\ln a = \text{Y-intercept}$$

$$\ln a = 1.6$$

$$a \approx e^{1.6}$$

$$a \approx 4.9530$$

$$a \approx 4.95$$

A2 (actual value of $a = 5 \pm 0.3$ and $b = 1.2 \pm 0.3$)

- (iii) It is discovered that one of the readings of the mass, m , is incorrect. Using your graph, estimate the correct value for this incorrect reading. [2]

When $t = 4$,

$$\ln m = -3.15$$

$$m = e^{-3.15}$$

$$m \approx 0.042852$$

$$m \approx 0.0429$$

M1 changing ln to exp

A1

NAME: _____ ()

CLASS: 4 ()

**SECONDARY FOUR
PRELIMINARY EXAMINATIONS 2025**

**ADDITIONAL MATHEMATICS****4049/02**

Paper 2 **MARKING SCHEME**

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3		10		Clarity / Logic	
4		11		Precision / Accuracy	
5				Total:	
6				90	
7					
Date:					

This paper consists of **18** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of } \Delta = \frac{1}{2} ab \sin C$$

1 (a) Show that $\frac{4}{\cot \theta - \tan \theta} = 2 \tan 2\theta$. [4]

(a)	$\begin{aligned} \text{LHS} &= \frac{4}{\cot \theta - \tan \theta} \\ &= \frac{4}{\frac{\cos \theta}{\sin \theta} - \frac{\sin \theta}{\cos \theta}} \\ &= \frac{4}{\frac{\cos^2 \theta - \sin^2 \theta}{\cos \theta \sin \theta}} \\ &= \frac{4 \cos \theta \sin \theta}{\cos^2 \theta - \sin^2 \theta} \div \frac{\cos^2 \theta}{\cos^2 \theta} \\ &= \frac{4 \tan \theta}{1 - \tan^2 \theta} \\ &= 2 \tan 2\theta \end{aligned}$	M1 for using changing tan and cot M1 M1 A1
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(b) Hence, solve the equation $\frac{4}{\cot 2x - \tan 2x} = 5$ for $0 < x < \pi$. [3]

(b)	$\begin{aligned} 2 \tan 4\theta &= 5 \\ \tan 4\theta &= \frac{5}{2} \\ \text{basic angle} &= 1.1903 \\ 4\theta &= 1.1903, \pi + 1.1903, 2\pi + 1.1903, 3\pi + 1.1903 \\ \theta &= 0.298, 1.08, 1.87, 2.65 \end{aligned}$	M1 M1 A1
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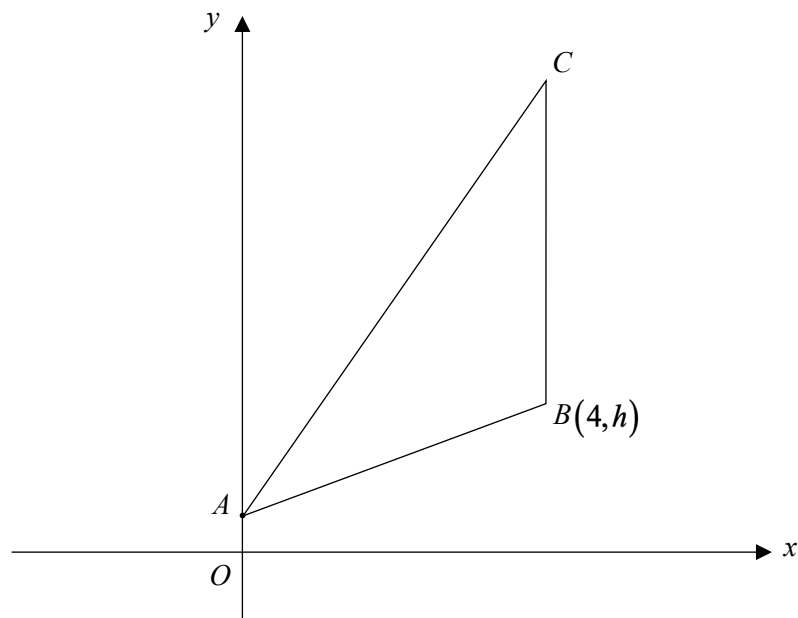
- 2 The expression $4x^3 + ax + b$, where a and b are constants, has a factor of $x - 1$, and a remainder of -9 when divided by $x + 2$.

(a) Find the value of a and of b .

[4]

Let $f(x) = 4x^3 + ax + b$ $f(1) = 0$ $4(1)^3 + a(1) + b = 0$ $4 + a + b = 0$ $a + b = -4 \text{-----}(1)$ $f(-2) = -9$ $4(-2)^3 + a(-2) + b = -9$ $-32 - 2a + b = -9$ $-2a + b = 23 \text{-----}(2)$ $(1) - (2):$ $3a = -27$ $a = -9$ Sub $a = -9$ into (1) $-9 + b = -4$ $b = 5$ $\therefore a = -9$ and $b = 5$	M1 for forming first equation M1 for forming second equation M1 for solving A1 for both answers
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- 3 The diagram shows an isosceles triangle ABC , where $AB = BC$ and the line segment BC is parallel to the y -axis. Point A lies on the y -axis and the equation of line AC is $y = 2x + 3$. Point B is $(4, h)$, where h is a positive constant.



- (a) Find the coordinates of A , B and C .

[4]

(a)	Since A is on the y-axis, $A(0, 3)$ Since BC is parallel to the y-axis, and x-coordinate of B is 4, x-coordinate of $C = 4$. $y = 2(4) + 3$ $= 11$ $C(4, 11)$ $\sqrt{4^2 + (h - 3)^2} = 11 - h$ $16 + (h - 3)^2 = (11 - h)^2$ $16 + h^2 - 6h + 9 = 121 - 22h + h^2$ $16h = 96$ $h = 6$ $B(4, 6)$	B1 B1 M1: form equation using length formula A1
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Given that the point D is such that $ABCD$ is a kite, and the line segment CD is parallel to the x -axis.

(b) Find the coordinates of D .

[4]

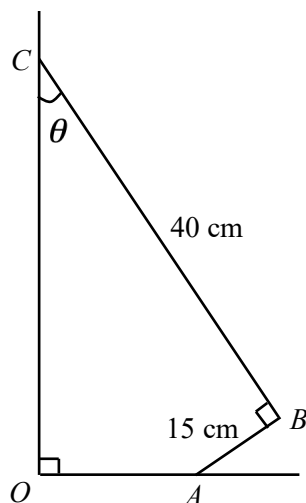
(b)	$\text{Gradient of } BD = -\frac{1}{2}$ $y = -\frac{1}{2}x + c$ $6 = -\frac{1}{2}(4) + c$ $c = 8$ $\text{Equation of } BD: y = -\frac{1}{2}x + 8$ Since CD is parallel to the x-axis, y-coordinate of $D = 11$. $11 = -\frac{1}{2}x + 8$ $3 = -\frac{1}{2}x$ $x = -6$ $D(-6, 11)$	M1: for finding gradient A1 M1: for subst $y = 11$ A1
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(c) Find the area of the kite $ABCD$.

[2]

(c)	$\text{Area} = \frac{1}{2} \begin{vmatrix} 0 & 4 & 4 & -6 & 0 \\ 3 & 6 & 11 & 11 & 3 \end{vmatrix}$ $= \frac{1}{2} [(0 + 44 + 44 - 18) - (12 + 24 - 66 + 0)]$ $= 50 \text{ units}^2$	M1 in anticlockwise A1
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- 4 In the diagram, it is given that $AB = 15$ cm, $BC = 40$ cm and $\angle OCB = \theta$. The vertical line OC is perpendicular to OA and AB is perpendicular to BC .



- (a) Show that $OC = 15\sin\theta + 40\cos\theta$. [2]

(i)	$\sin\theta = \frac{AB}{15}$ $\cos\theta = \frac{BC}{40}$ $OC = AB\sin\theta + BC\cos\theta$ $= 15\sin\theta + 40\cos\theta$	B1 B1
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- (b) Express OC in the form $R\sin(\theta + \alpha)$, where R is a positive constant and α is an acute angle in degrees. [3]

(ii)	$R = 40^2 + 15^2 = \sqrt{1825}$ $\tan\alpha = \frac{40}{15}$ $\alpha = 69.444\dots$ $OC = 42.7\sin(\theta + 69.4^\circ)$	M1 M1 A1
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- (c) Given that OC is at the maximum, determine the shortest distance of B to OC . [3]

(iii)	$\max OC = 42.7 \text{ cm when } \theta + 69.444\dots^\circ = 90^\circ$ $\theta = 20.556^\circ$ $\text{shortest distance} = 40\sin(20.556^\circ)$ $= 14.0\text{cm}$	M1 M1 A1
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5 (i) Show $\frac{d}{dx}[\ln(\tan 3x)] = \frac{k}{\sin 6x}$, stating the value of the constant k . [4]

(i)	$\begin{aligned}\frac{d}{dx}[\ln(\tan 3x)] &= \frac{1}{\tan 3x}(\sec^2 3x)(3) \\ &= \frac{3 \cos 3x}{\sin 3x \cos^2 3x} \\ &= \frac{3}{\sin 3x \cos 3x} \\ &= \frac{3(2)}{2 \sin 3x \cos 3x} \\ &= \frac{6}{\sin 6x}\end{aligned}$ $k = 6$	M1 – differentiate correctly M1 – simplify trigo functions M1 – correct application of sine double angle formula A1
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(ii) Differentiate $\ln \sqrt{\frac{7x}{x^2-1}}$ with respect to x . [3]

(ii)	$\begin{aligned}\frac{d}{dx}\left(\ln \sqrt{\frac{7x}{x^2-1}}\right) &= \frac{d}{dx}\left(\frac{1}{2} \ln \frac{7x}{x^2-1}\right) \\ &= \frac{d}{dx}\left(\frac{1}{2}[\ln 7x - \ln(x^2-1)]\right) \\ &= \frac{1}{2}\left(\frac{7}{7x}\right) - \frac{1}{2}\left(\frac{2x}{x^2-1}\right) \\ &= \frac{1}{2}\left(\frac{x^2-1-2x^2}{x(x^2-1)}\right) \\ &= -\frac{x^2+1}{2x(x^2-1)}\end{aligned}$	M1 – use logarithm laws to simplify M1 – differentiate $\ln 7x$ and $\ln(x^2-1)$ A1
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6 The curve $y = f(x)$ is such that $f''(x) = 2e^x + e^{-2}$.

The y -axis is a normal to the curve at the point Q , where $y = 1$.

(a) Show that Q is a stationary point.

[2]

(a)	The x -axis is perpendicular to the y -axis. Since the y -axis is a normal, the x -axis is parallel to a tangent at $y = 1$. Since the gradient of the x -axis is 0, $\frac{dy}{dx} = 0$ at $y = 1$. Therefore, there is a stationary point.	B1: finding $\frac{dy}{dx}$ B1: stating why gradient = 0
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(b) Hence, find an expression for $f(x)$.

[6]

(b)	$f'(x) = \int 2e^x + e^{-2x} dx$ $= 2e^x + \frac{e^{-2x}}{-2} + c$ $= 2e^x - \frac{1}{2e^{2x}} + c$ At $x = 0$, $f'(x) = 0$, $2 - \frac{1}{2} + c = 0$ $c = -\frac{3}{2}$ $f'(x) = 2e^x - \frac{1}{2e^{2x}} - \frac{3}{2}$ $f(x) = \int 2e^x - \frac{1}{2e^{2x}} - \frac{3}{2} dx$ $= 2e^x - \frac{1}{2} \left(\frac{e^{-2x}}{-2} \right) - \frac{3}{2}x + c$ $= 2e^x + \frac{1}{4e^{2x}} - \frac{3}{2}x + c$ At $x = 0$, $y = 1$, $1 = 2 + \frac{1}{4} + c$ $c = -\frac{5}{4}$ $f(x) = 2e^x + \frac{1}{4e} - \frac{3}{2}x - \frac{5}{4}$	M1: for correct integration M1 A1 M1 M1 A1
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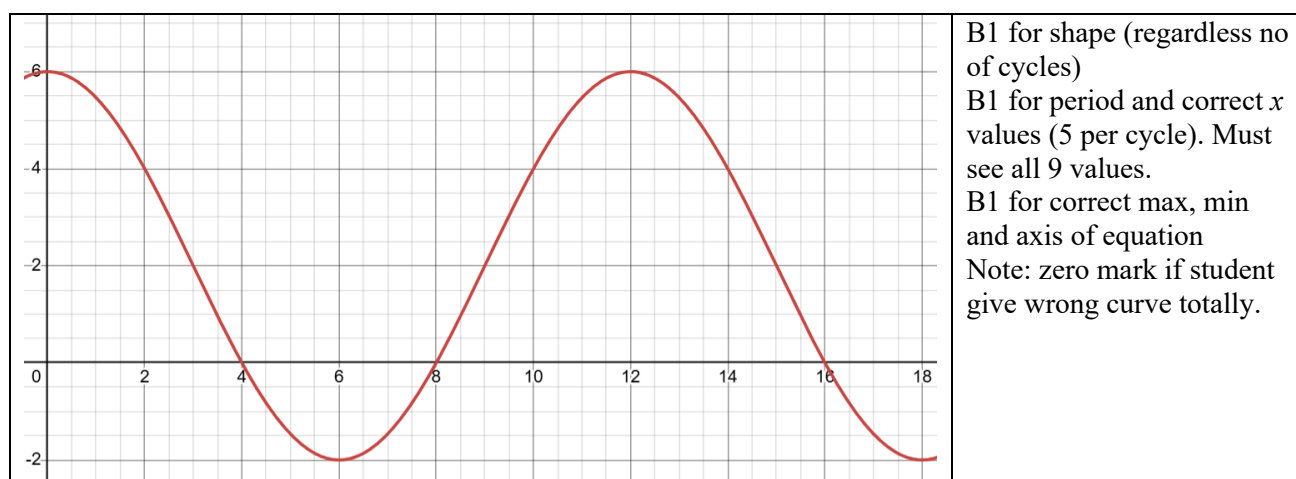
- 7 A waterwheel rotates at 1 revolution per 12 seconds. Initially, Point A , marked on the highest point of the rim of the wheel is 6 m above the water surface. It reaches 2 m below the water surface 6 seconds later.

The motion of point A can be modelled using the trigonometric equation $h = b \cos(kt) + 2$, where the height of point A above the water surface, h , is a function of time t , in seconds.

- (i) Show that $b = 4$ and $k = \frac{\pi}{6}$. [2]

$b = \frac{6+2}{2} = 4$	B1
$k = \frac{2\pi}{12} = \frac{\pi}{6}$	B1

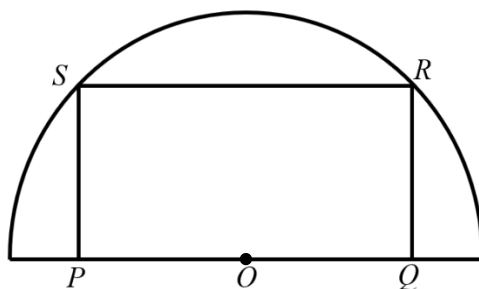
- (ii) On the axes below, sketch the graph of h , for $0 \leq t \leq 18$. [3]



- (iii) A similar waterwheel at a different location has a point, B , marked on its rim. The motion of point B is modelled by the equation $h_B = 4 \cos(\frac{\pi}{6}t) + 4$. By drawing a suitable line on the same axis above, determine the number of time(s) when point B touches the water surface from $0 \leq t \leq 18$. [2]

2 times	M1 for drawing line of $y = -2$
	A1

- 8 In the figure, $PQRS$ is a rectangle which fits inside a semicircle of radius 8 cm and centre O .



If $PQ = x$ cm and $QR = y$ cm,

- (a) Show that the area of the rectangle, $A \text{ cm}^2$, is given by $A = \frac{x}{2}\sqrt{256 - x^2}$. [3]

(a)	$A = xy$ $8^2 = y^2 + \left(\frac{x}{2}\right)^2$ $64 = y^2 + \frac{x^2}{4}$ $y = \sqrt{\frac{256 - x^2}{4}}$ $A = x \left(\sqrt{\frac{256 - x^2}{4}} \right)$ $A = \frac{x}{2} \sqrt{256 - x^2}$	M1 – form Pythagoras Theorem M1 – make y the subject A1
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- (b) The maximum area is obtained when the value of x , which can vary, gives a stationary value of A . Find the maximum value of A . [6]

(b)

$$A = \frac{x}{2} \sqrt{256 - x^2}$$

$$\frac{dA}{dx} = \left(\frac{1}{2} x \right) \left(\frac{1}{2} \right) (256 - x^2)^{-\frac{1}{2}} (-2x) + (256 - x^2)^{\frac{1}{2}} \left(\frac{1}{2} \right)$$

$$= \frac{1}{2} (256 - x^2)^{-\frac{1}{2}} [-x^2 + 256 - x^2]$$

$$= \frac{-2x^2 + 256}{2\sqrt{256 - x^2}}$$

$$\frac{dA}{dx} = 0$$

$$\frac{-2x^2 + 256}{2\sqrt{256 - x^2}} = 0$$

$$-2x^2 + 256 = 0$$

$$x^2 = 128$$

$$x = \sqrt{128}$$

$$A = \frac{\sqrt{128}}{2} \sqrt{256 - (\sqrt{128})^2}$$

$$A = 64$$

x	$\sqrt{128}^-$	$\sqrt{128}$	$\sqrt{128}^+$
Sign of $\frac{dA}{dx}$	-	0	+
Sketch of tangent			

By first derivative testing, A is a maximum.

M1 – differentiate correctly

M1 – equate derivative to 0

M1 – find x

A1

M1

A1

(b) Find the area of the shaded region. Leave your answer in the form $a + b \ln 2$. [6]

(b)	$\frac{8}{2+3x} = -\frac{3}{2}x + 4$ $\frac{16}{2+3x} = -3x + 8$ $16 = -6x - 9x^2 + 16 + 24x$ $9x^2 - 18x = 0$ $x(x-2) = 0$ $x = 0 \quad \text{or} \quad 2$ At $x = 2$, $y = -\frac{3}{2}(2) + 4$ $= 1$ $\text{Area} = \frac{1}{2}(4+1)(2) - \int_0^2 \frac{8}{2+3x} dx$ $= 5 - \frac{8}{3} [\ln(2+3x)]_0^2$ $= 5 - \frac{8}{3} [\ln 8 - \ln 2]$ $= 5 - \frac{16}{3} \ln 2$	M1: for equating the equations A1: for coordinates of other intersection point M1: for area of trapezium / integrating line minus integrating curve M1: for integration M1: for definite integrals A1
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- 10** A particle travels in a straight line such that at time t seconds after leaving a point A , which has a displacement of 2 metres from a fixed point O , its velocity, $v \text{ ms}^{-1}$ is given by
- $$v = 3t^2 - 21t + 18.$$

Find,

- (a)** the minimum velocity of the particle,

[3]

(a)	$\frac{dv}{dt} = 6t - 21$ $6t - 21 = 0$ $t = \frac{21}{6} \text{ s}$ $v = 3\left(\frac{21}{6}\right)^2 - 21\left(\frac{21}{6}\right) + 18$ $= -\frac{75}{4} \text{ ms}^{-1}$ $\frac{d^2v}{dt^2} = 6 > 0$ Therefore, $v = -\frac{75}{4} \text{ ms}^{-1}$ is the minimum velocity.	M1: for finding t (Award even if have careless mistake) A1 B1: for checking max/min (Award even if value of v is wrong)
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- (b)** the time(s) at which the particle is instantaneously at rest,

[1]

(b)	$3t^2 - 21t + 18 = 0$ $t^2 - 7t + 6 = 0$ $(t - 6)(t - 1) = 0$ $t = 1 \text{ or } 6$	B1
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(c) the total distance travelled in the first 7 seconds.

[6]

$$s = \int 3t^2 - 21t + 18 \, dt$$

$$= t^3 - \frac{21}{2}t^2 + 18t + c$$

At $t = 0$, $s = 2$, therefore, $c = 2$

$$s = t^3 - \frac{21}{2}t^2 + 18t + 2$$

At $t = 1$,

$$s = 1^3 - \frac{21}{2}(1)^2 + 18(1) + 2$$

$$= 10.5m$$

At $t = 6$,

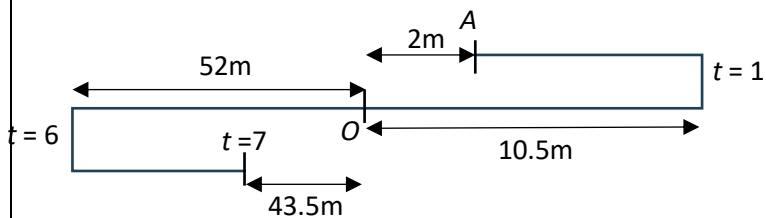
$$s = 6^3 - \frac{21}{2}(6)^2 + 18(6) + 2$$

$$= -52m$$

At $t = 7$,

$$s = 7^3 - \frac{21}{2}(7)^2 + 18(7) + 2$$

$$= -43.5m$$



$$\text{Total Distance} = (10.5 - 2) + 10.5 + 52 + (52 - 43.5)$$

$$= 79.5m$$

B1: for getting s

M1

M1

M1

M1

A1

- 11 (a)** Given the function $y = \frac{m-2x}{x^2-3}$, find $\frac{dy}{dx}$. [3]

$y = \frac{m-2x}{x^2-3}$ $\frac{dy}{dx} = \frac{(x^2-3)(-2) - (m-2x)(2x)}{(x^2-3)^2}$ $= \frac{-2x^2 + 6 - 2mx + 4x^2}{(x^2-3)^2}$ $= \frac{2x^2 + 6 - 2mx}{(x^2-3)^2}$	M2 – finding numerator and denominator $\frac{dy}{dx}$ A1
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- (b)** Hence, find the range of values of m such that the function $y = \frac{m-2x}{x^2-3}$ is always increasing. [4]

Since $\frac{2x^2 + 6 - 2mx}{(x^2-3)^2} > 0$ and $(x^2-3)^2 > 0$, $\therefore 2x^2 - 2mx + 6 > 0$ $2x^2 - 2mx + 6 = 2(x^2 - mx + 3)$ $= 2 \left[\left(x - \frac{m}{2} \right)^2 - \left(\frac{m}{2} \right)^2 + 3 \right]$ $= 2 \left(x - \frac{m}{2} \right)^2 - \frac{m^2}{2} + 6$ For $2x^2 - 2mx + 6 > 0$, $-\frac{m^2}{2} + 6 > 0$ $\frac{m^2}{2} < 6$ $m^2 < 12$ $m^2 - 12 < 0$ $(m + \sqrt{12})(m - \sqrt{12}) < 0$ $-2\sqrt{3} < m < 2\sqrt{3}$	M1 – stating $\frac{dy}{dx} > 0$ or wrong expression > 0 M1 – completing the square method M1 – stating $-\frac{m^2}{2} + 6 > 0$ A1
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Alternative Solution Since $\frac{2x^2 + 6 - 2mx}{(x^2 - 3)^2} > 0$ and $(x^2 - 3)^2 > 0$, $\therefore 2x^2 - 2mx + 6 > 0$ $2x^2 - 2mx + 6 = 0$ discriminant < 0 $(-2m)^2 - 4(2)(6) < 0$ $4m^2 - 48 < 0$ $m^2 - 12 < 0$ $(m + \sqrt{12})(m - \sqrt{12}) < 0$ $-2\sqrt{3} < m < 2\sqrt{3}$	M1 – stating $\frac{dy}{dx} > 0$ or wrong expression > 0 M1 – finding discriminant M1 – stating discriminant < 0 A1
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END OF PAPER