

|                           |                  |                            |                    |
|---------------------------|------------------|----------------------------|--------------------|
| <b>Year</b>               | 2025             | <b>Level &amp; Stream</b>  | 4E5N               |
| <b>Type of Assessment</b> | Preliminary Exam | <b>Subject &amp; Paper</b> | Additional Math P1 |

| <b>Qns</b> | <b>Working</b>                                                                                                                                                                                                                                                                                                         |
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| 1          | $4x^2 - 6xy - 6y = 4 \text{ ----(1)}$ $y - x = 4 \text{ -----(2)}$ <p>Sub (2) into (1):</p> $4x^2 - 6x(4 + x) - 6(4 + x) = 4$ $4x^2 - 24x - 6x^2 - 24 - 6x = 4$ $-2x^2 - 30x - 28 = 0$ $x^2 + 15x + 14 = 0$ $(x+1)(x+14) = 0$ $x = -14 \text{ or } x = -1$ $y = -10 \text{ or } y = 3$                                 |
| 2          | $9 \sin x = -2 \sec x$ $9 \sin x = -\frac{2}{\cos x}$ $\sin x \cos x = -\frac{2}{9}$ $2 \sin x \cos x = -\frac{4}{9}$ $\sin 2x = -\frac{4}{9}$ $\text{Basic angle} = \sin^{-1}\left(\frac{4}{9}\right)$ $= 0.46055$ $2x = -0.46055, -(\pi - 0.46055)$ $= -0.46055 \text{ or } -2.68104$ $x = -0.230 \text{ or } -1.34$ |

| Qns | Working                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
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| 3a  | <p>Principal range of <math>\cos^{-1} x</math> is <math>0 \leq \cos^{-1} x \leq \pi</math>,</p> <p>hence principal value of <math>\cos^{-1} \left( -\frac{\sqrt{2}}{2} \right)</math> is <math>\frac{3\pi}{4}</math></p> <p>instead of <math>-\frac{\pi}{4}</math>.</p>                                                                                                                                                                                                                                                                                                                                    |
| 3b  | <p>L.H.S: <math>\cot A + \tan A = \frac{1}{\tan A} + \tan A</math></p> $= \frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}$ $= \frac{\cos^2 A + \sin^2 A}{\sin A \cos A}$ $= \frac{1}{\sin A \cos A}$ $= \sec A \operatorname{cosec} A = \text{RHS (shown)}$ <p><b>OR</b></p> <p>RHS:</p> $\sec A \operatorname{cosec} A = \frac{1}{\cos A} \times \frac{1}{\sin A}$ $= \frac{1}{\cos A \sin A}$ $= \frac{\sin^2 A + \cos^2 A}{\cos A \sin A}$ $= \frac{\sin^2 A}{\cos A \sin A} + \frac{\cos^2 A}{\cos A \sin A}$ $= \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$ $= \tan A + \cot A = \text{LHS (shown)}$ |

| Qns | Working                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
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| 4   | <p>By long division,</p> $\frac{2x^3 + 4}{x(x+2)^2} = \frac{2x^3 + 4}{x^3 + 4x^2 + 4x}$ $= 2 + \frac{-8x^2 - 8x + 4}{x(x+2)^2}$ $\frac{-8x^2 - 8x + 4}{x(x+2)^2} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$ $-8x^2 - 8x + 4 = A(x+2)^2 + Bx(x+2) + Cx$ <p>Let <math>x = 0</math>:</p> $4 = 4A$ $A = 1$ <p>Let <math>x = -2</math>:</p> $-12 = -2C$ $C = 6$ <p>Let <math>x = 1</math>:</p> $-8 - 8 + 4 = 9 + 3B + 6$ $B = -9$ $\therefore \frac{2x^3 + 4}{x(x+2)^2} = 2 + \frac{1}{x} - \frac{9}{x+2} + \frac{6}{(x+2)^2}$ |
| 5a  | $f'(x) = \frac{(x^2 + 3) - 2x(x+a)}{(x^2 + 3)^2}$ $= \frac{-x^2 - 2ax + 3}{(x^2 + 3)^2}$                                                                                                                                                                                                                                                                                                                                                                                                                                          |
| 5b  | $\frac{-x^2 - 2ax + 3}{(x^2 + 3)^2} > 0, \text{ for } b < x < 1$ <p>Since <math>(x^2 + 3)^2 &gt; 0</math>,</p> $-x^2 - 2ax + 3 > 0$ $x^2 + 2ax - 3 < 0$ <p>Given that <math>b &lt; x &lt; 1</math></p> $(x - b)(x - 1) < 0$ $x^2 - bx - x + b < 0$ <p>By comparing:</p> $b = -3$ $2a = -b - 1$ $2a = 3 - 1$ $a = 1$                                                                                                                                                                                                               |

| Qns | Working                                                                                                                                                                                                                                                                                                                                               |
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| 6   | $y = (k-6)x^2 - 8x + k$ <p>Discriminant <math>= (-8)^2 - 4(k-6)(k)</math></p> $= 64 - 4k^2 + 24k$ <p>For curve that does not intersect the <math>x</math>-axis,<br/>Discriminant <math>&lt; 0</math></p> $k^2 - 6k - 16 > 0$ $(k+2)(k-8) > 0$ $k < -2 \text{ or } k > 8$ <p>The curve has a minimum point:</p> $k - 6 > 0$ $k > 6$ $\therefore k > 8$ |
| 7a  | $\frac{d}{dx}[(x-1)e^{-x}] = e^{-x} - (x-1)e^{-x}$ $= 2e^{-x} - xe^{-x}$                                                                                                                                                                                                                                                                              |
| 7b  | $\int 2e^{-x} - xe^{-x} dx = (x-1)e^{-x} + c$ $\int 2e^{-x} dx - \int xe^{-x} dx = (x-1)e^{-x} + c$ $-\int xe^{-x} dx = -\int 2e^{-x} dx + (x-1)e^{-x} + c$ $\int xe^{-x} dx = -2e^{-x} - (x-1)e^{-x} + c$ $= -2e^{-x} - xe^{-x} + e^{-x} + c$ $= -e^{-x} - xe^{-x} + c$                                                                              |

| Qns | Working                                                                                                                                                                                                                                                                                                                                   |
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| 8a  | $\frac{dr}{dt} = -\frac{2}{(t+1)^2}$ $r = -\int \frac{2}{(t+1)^2} dt$ $= -\int 2(t+1)^{-2} dt$ $= 2(t+1)^{-1} + c$ $r = \frac{2}{t+1} + c$ <p>At <math>r = 4</math> and <math>t = 0</math>,</p> $4 = 2 + c$ $c = 2$ $r = \frac{2}{t+1} + 2$                                                                                               |
| 8b  | $A = \pi r^2$ $\frac{dA}{dr} = 2\pi r$ <p>At <math>r = 2.6</math>, <math>\frac{dA}{dr} = 5.2\pi</math>,</p> $2.6 = \frac{2}{t+1} + 2$ $0.6 = \frac{2}{t+1}$ $t = \frac{7}{3}$ $\frac{dr}{dt} = -\frac{2}{\left(\frac{7}{3}+1\right)^2} = -0.18$ $\frac{dA}{dt} = 5.2\pi \times -0.18$ $\frac{dA}{dt} = -0.936\pi \text{ cm}^2/\text{min}$ |

| Qns | Working                                                                                                                                                                                                                                                                                                                                                                                                      |
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| 9a  | $y = kx + 6 \text{ ---- (1)}$ $2x^2 - xy = 3 \text{ -----(2)}$ <p>Sub (1) into (2):</p> $2x^2 - x(kx + 6) = 3$ $2x^2 - kx^2 - 6x - 3 = 0$ $(2 - k)x^2 - 6x - 3 = 0$ <p>Discriminant</p> $= (-6)^2 - 4(2 - k)(-3)$ $= 60 - 12k$ <p>For tangent,</p> $60 - 12k = 0$ $k = 5$                                                                                                                                    |
| 9b  | $(4 - 2\sqrt{2})x + \sqrt{2} = 3x\sqrt{2} - 1$ $4x - 2x\sqrt{2} + \sqrt{2} = 3x\sqrt{2} - 1$ $\sqrt{2} + 1 = 3x\sqrt{2} + 2x\sqrt{2} - 4x$ $\sqrt{2} + 1 = x(5\sqrt{2} - 4)$ $x = \frac{\sqrt{2} + 1}{5\sqrt{2} - 4} \times \frac{5\sqrt{2} + 4}{5\sqrt{2} + 4}$ $= \frac{5(2) + 4\sqrt{2} + 5\sqrt{2} + 4}{(5\sqrt{2})^2 - 4^2}$ $= \frac{9\sqrt{2} + 14}{50 - 16}$ $= \frac{9}{34}\sqrt{2} + \frac{7}{17}$ |

| Qns | Working                                                                                                                                                                                                                                                                                                                                                  |
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| 10a | $\text{Area of } \Delta PQR = \frac{1}{2} \begin{vmatrix} 1 & 2 & 0 & 1 \\ 0 & 3 & 2 & 0 \end{vmatrix}$ $= \frac{1}{2} [3 + 4 - 2]$ $= 2.5 \text{ units}^2$                                                                                                                                                                                              |
| 10b | $m_{PR} = \frac{3-0}{2-1}$ $= 3$ $m_{QS} = -\frac{1}{3}$ <p>Equation of QS:</p> $y - 2 = -\frac{1}{3}(x - 0)$ $y = -\frac{1}{3}x + 2 \text{ --- (1)}$ $x - y = 5 \text{ --- (2)}$ <p>Sub (1) into (2):</p> $x + \frac{1}{3}x - 2 = 5$ $x = \frac{21}{4}$ $y = \frac{1}{4}$ <p>coordinates of S = <math>\left(\frac{21}{4}, \frac{1}{4}\right)</math></p> |

| Qns | Working                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
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| 11a | $2^x + 4^{x-1} = 24$ $2^x + (4^x)(4^{-1}) = 24$ $2^x + \frac{2^{2x}}{4} = 24$ <p>Let <math>y = 2^x</math>,</p> $y + \frac{y^2}{4} = 24$ $4y + y^2 = 96$ $y^2 + 4y - 96 = 0$ $(y - 8)(y + 12) = 0$ $y = 8 \quad \text{or} \quad y = -12$ $2^x = 8 \quad \text{or} \quad 2^x = -12 \quad (\text{Rejected since } 2^x > 0)$ $2^x = 2^3$ $x = 3$                                                                                                                      |
| 11b | $\log_{\sqrt{3}}(x - 1) = \log_3(x + 5)$ $\frac{\log_3(x - 1)}{\log_3 \sqrt{3}} = \log_3(x + 5)$ $\frac{\log_3(x - 1)}{\log_3 3^{\frac{1}{2}}} = \log_3(x + 5)$ $2 \log_3(x - 1) = \log_3(x + 5)$ $\log_3(x - 1)^2 = \log_3(x + 5)$ $(x - 1)^2 = x + 5$ $x^2 - 2x + 1 - x - 5 = 0$ $x^2 - 3x - 4 = 0$ $(x + 1)(x - 4) = 0$ $x + 1 = 0$ $x = -1 \quad [\text{N.A. as } \log_{\sqrt{3}}(x - 1) \text{ will be undefined}]$ <p>or <math>x - 4 = 0</math></p> $x = 4$ |

| Qns | Working                                                                                                                                                                                                                                                                              |
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| 12a | $4x - 4y = 3$ $y = x - \frac{3}{4}$ $m \text{ of normal} = 1$ $m \text{ of tangent} = -1$ $\text{At } x = \frac{1}{2} \text{ and } \frac{dy}{dx} = -1,$ $6\left(\frac{1}{2}\right) - \frac{1}{k\left(\frac{1}{2}\right)^3} = -1$ $3 - \frac{8}{k} = -1$ $k = 2 \quad (\text{shown})$ |
| 12b | $\text{At stat points,}$ $6x - \frac{1}{2x^3} = 0$ $6x = \frac{1}{2x^3}$ $x^4 = \frac{1}{12}$ $x = \pm \sqrt[4]{\frac{1}{12}}$ $x = -0.537 \text{ or } 0.537$                                                                                                                        |
| 12c | $\frac{d^2y}{dx^2} = 6 + \frac{3}{2x^4}$ $6 > 0$ $\frac{3}{2x^4} > 0$ $\therefore \frac{d^2y}{dx^2} = 6 + \frac{3}{2x^4} > 0$ $\text{Since } \frac{d^2y}{dx^2} = 6 + \frac{3}{2x^4} > 0$ <p>the gradient has no turning point.</p>                                                   |

| Qns | Working                                                                                                                                                                                                                                                                                       |
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| 13a | $C = 1.2n^2 - 14.4n + 53.7$ <p>At <math>n = 0</math>, <math>C = \\$53.7</math> thousands</p>                                                                                                                                                                                                  |
| 13b | $C = 1.2n^2 - 14.4n + 53.7$ $= 1.2[n^2 - 12n + 44.75]$ $= 1.2[(n-6)^2 - 6^2 + 44.75]$ $= 1.2(n-6)^2 + 10.5$                                                                                                                                                                                   |
| 13c | <p>For 600 pairs of running shoes produced, the minimum cost of 10.5 thousand dollars will be incurred.</p>                                                                                                                                                                                   |
| 13d | $1.2(n-6)^2 + 10.5 = 50$ $1.2(n-6)^2 = 39.5$ $(n-6)^2 = \frac{39.5}{1.2}$ $n-6 = \pm 5.7373$ $n = 11.7373 \text{ or } 0.2627$ <p>Maximum number of pairs of running shoes for which the cost is at most 50 thousand dollars<br/> <math>= 11.7373</math><br/> <math>= 11.7</math> hundreds</p> |

| Qns | Working                                                                                                                                                                                                                                                                                                                                                                              |
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| 14a | $m_{AB} = 1$<br>Equation of $AB$ :<br>$y + 2 = x + 4$<br>$y = x + 2 \text{ --- (1)}$<br>$y = -x + 4 \text{ --- (2)}$<br>$(1) = (2) : x + 2 = -x + 4$<br>$2x = 2$<br>$x = 1$<br>$y = 3$<br>Coordinates of $A = (1, 3)$                                                                                                                                                                |
| 14b | $\text{Radius} = \sqrt{(1+4)^2 + (3+2)^2}$ $= \sqrt{50} \text{ units}$ Equation of circle:<br>$(x+4)^2 + (y+2)^2 = 50$                                                                                                                                                                                                                                                               |
| 14c | Let the coordinates of the other end of the diameter passes through $A = C(x, y)$ .<br>Since $AC$ is the diameter, $B$ is the midpoint of $AC$ .<br>$\left( \frac{x+1}{2}, \frac{y+3}{2} \right) = (-4, -2)$ $\frac{x+1}{2} = -4, \quad \frac{y+3}{2} = -2$ $x = -9, y = -7$ Equation of another tangent which is parallel to $y = -x + 4$ :<br>$y + 7 = -1(x + 9)$<br>$y = -x - 16$ |

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| Year               | 2025             | Level & Stream  | 4E                 |
| Type of Assessment | Preliminary Exam | Subject & Paper | Additional Math P2 |

| Qns               | Working                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |                                                                                     |                                                                                     |         |     |                 |     |   |     |                   |                                                                                     |                                                                                     |                                                                                     |
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| 1a                | $N(x) = x^2 \ln\left(\frac{2}{x}\right)$ $\text{let } u = x^2 \text{ and } v = \ln\left(\frac{2}{x}\right)$ $\frac{du}{dx} = 2x \qquad v = \ln 2 - \ln x$ $\frac{dv}{dx} = -\frac{1}{x}$ $N'(x) = x^2 \cdot \left(-\frac{1}{x}\right) + \ln\left(\frac{2}{x}\right) \cdot 2x$ $= -x + 2x \ln\left(\frac{2}{x}\right)$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |                                                                                     |                                                                                     |         |     |                 |     |   |     |                   |                                                                                     |                                                                                     |                                                                                     |
| b                 | <p>stationary point <math>N'(x) = 0</math></p> $-x + 2x \ln\left(\frac{2}{x}\right) = 0$ $x \left(-1 + 2 \ln\left(\frac{2}{x}\right)\right) = 0$ $x = 0 \text{ (rej) } \quad \text{or} \quad -1 + 2 \ln\left(\frac{2}{x}\right) = 0$ $2 \ln\left(\frac{2}{x}\right) = 1$ $\ln\left(\frac{2}{x}\right) = \frac{1}{2}$ $e^{0.5} = \frac{2}{x}$ $x = 2 \div e^{0.5}$ $x = 1.21306$ <table border="1"><tr><td><math>x</math></td><td>1.1</td><td>1.21306</td><td>1.3</td></tr><tr><td>Sign of <math>N'(x)</math></td><td>+ve</td><td>0</td><td>-ve</td></tr><tr><td>Sketch of tangent</td><td></td><td></td><td></td></tr></table> <p><math>\therefore</math> max absorption rate at <math>x = 1.21</math> (3 s.f.)</p> | $x$                                                                                 | 1.1                                                                                 | 1.21306 | 1.3 | Sign of $N'(x)$ | +ve | 0 | -ve | Sketch of tangent |  |  |  |
| $x$               | 1.1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | 1.21306                                                                             | 1.3                                                                                 |         |     |                 |     |   |     |                   |                                                                                     |                                                                                     |                                                                                     |
| Sign of $N'(x)$   | +ve                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | 0                                                                                   | -ve                                                                                 |         |     |                 |     |   |     |                   |                                                                                     |                                                                                     |                                                                                     |
| Sketch of tangent |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |  |  |         |     |                 |     |   |     |                   |                                                                                     |                                                                                     |                                                                                     |

| Qns | Working                                                                                                                                                                                                                                                                  |
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| 2a  | $T = 22 + Ae^{kt}$ <p>when <math>t = 0</math>, <math>T = 90</math></p> $90 = 22 + Ae^{k(0)}$ $90 = 22 + A$ $A = 68$                                                                                                                                                      |
| b   | <p>when <math>t = 3.5</math>, <math>T = 79</math></p> $79 = 22 + 68e^{k(3.5)}$ $57 = 68e^{k(3.5)}$ $\frac{57}{68} = e^{k(3.5)}$ $\ln\left(\frac{57}{68}\right) = k(3.5)$ $k = \ln\left(\frac{57}{68}\right) \div 3.5$ $k = -0.050416$ $k = -0.0504 \text{ (3 s.f.)}$     |
| c   | $T = 22 + 68e^{-0.050416t}$ <p>when <math>t = 10</math>,</p> $T = 22 + 68e^{-0.050416(10)}$ $= 63.0728$ <p>Since <math>63.0728^\circ\text{C}</math> is between <math>55^\circ\text{C}</math> and <math>70^\circ\text{C}</math>, the coffee is suitable to be served.</p> |
| d   | <p>After a long time, <math>T</math> approaches <math>22^\circ\text{C}</math>.</p>                                                                                                                                                                                       |

| Qns | Working                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
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| 3a  | $\cos \theta = \frac{ED}{14}$ $ED = 14 \cos \theta$ <p>Let <math>G</math> be the point on <math>FD</math> such that <math>BG \perp FD</math></p> $\sin \theta = \frac{EG}{3}$ $EG = 3 \sin \theta$ $FD = 14 \cos \theta + 3 \sin \theta + 6 \text{ (shown)}$                                                                                                                                                                                                |
| b   | $R = \sqrt{14^2 + 3^2}$ $= \sqrt{205}$ $\tan \alpha = \frac{3}{14}$ $\alpha = 12.0947$ $FD = \sqrt{205} \cos(\theta - 12.1^\circ) + 6 \text{ (3 s.f.)}$                                                                                                                                                                                                                                                                                                     |
| c   | $\sqrt{205} \cos(\theta - 12.0947^\circ) + 6 = 12$ $\cos(\theta - 12.0947^\circ) = \frac{6}{\sqrt{205}}$ $\theta - 12.0947^\circ = 65.2248^\circ$ $\theta = 77.3195^\circ$ $\theta = 77.3^\circ \text{ (1 d.p.)}$                                                                                                                                                                                                                                           |
| d   | <p>Maximum <math>\sqrt{205} \cos(\theta - 12.0947^\circ) + 6</math></p> $= \sqrt{205} + 6$ $= 20.317$ <p><math>\therefore</math> it is possible for <math>FD</math> to be 15 cm</p> <p>Or</p> $\sqrt{205} \cos(\theta - 12.0947^\circ) + 6 = 15$ $\cos(\theta - 12.0947^\circ) = \frac{9}{\sqrt{205}}$ $\theta - 12.0947^\circ = 51.054^\circ$ $\theta = 63.1^\circ \text{ (1 d.p.)}$ <p><math>FD</math> is 15 cm when <math>\theta = 63.1^\circ</math></p> |

| Qns | Working |
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| 4a  |         |



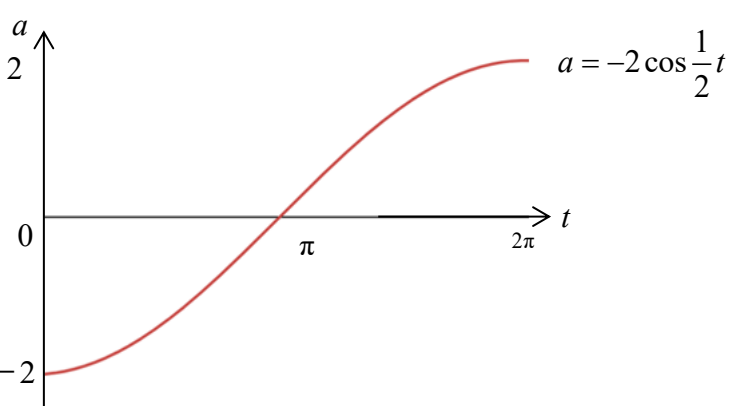
| Qns | Working                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
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| c   | $hx^3 - 7x^2 + kx - 2 = (3x^2 - 5x - 2) \times \text{Quotient}$ $hx^3 - 7x^2 + kx - 2 = (3x + 1)(x - 2) \times \text{Quotient}$ $\text{sub } x = -\frac{1}{3}, h\left(-\frac{1}{3}\right)^3 - 7\left(-\frac{1}{3}\right)^2 + k\left(-\frac{1}{3}\right) - 2 = 0$ $-\frac{1}{27}h - \frac{1}{3}k = \frac{25}{9} \text{-----(1)}$ $\text{sub } x = 2, h(2)^3 - 7(2)^2 + k(2) - 2 = 0$ $8h + 2k = 30 \text{-----(2)}$ <p>From (2), <math>4h + k = 15</math></p> $k = 15 - 4h \text{-----(3)}$ <p>sub (3) into (1), <math>-\frac{1}{27}h - \frac{1}{3}(15 - 4h) = \frac{25}{9}</math></p> $-\frac{1}{27}h - 5 + \frac{4}{3}h = \frac{25}{9}$ $\frac{35}{27}h = \frac{70}{9}$ $h = 6$ <p>sub <math>h = 6</math> into (3), <math>k = 15 - 4(6)</math></p> $k = -9$ |

| Qns | Working                                                                                                                                                                                                                                                                                                                                                                                 |
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| 6a  | $AB = AE$ (tangent from external point)<br>$AC = AD$ (tangent from external point)<br>$\frac{AB}{AC} = \frac{AE}{AD}$ (ratio of corresponding sides are equal)<br>$\angle BAE = \angle CAD$ (common angle)<br>$\triangle ABE$ is similar to $\triangle ACD$ (ratio of 2 pairs of corresponding sides and a pair of included $\angle$ s are equal)                                       |
| b   | $\angle ABE = \angle ACD$ ( $\triangle ABE$ is similar to $\triangle ACD$ )<br>$\angle ACD = \angle ADC$ (base $\angle$ of isosceles $\triangle$ )<br>$\angle EBC = 180^\circ - \angle ABE$ (adjacent $\angle$ s on a straight line)<br>Since $\angle ADC + \angle EBC = 180^\circ$ ,<br>by the converse of angles in opposite segments, a circle can be drawn to pass through $BEDC$ . |
| c   | Let $\angle CBF = x$ ,<br>$\angle BEF = x$ (alternate segment thm)<br>$BE \parallel CD$ since $\triangle ABE$ is similar to $\triangle ACD$<br>$\angle ECD = x$ (alternate angles, $BE \parallel CD$ )<br>$\angle EDF = x$ (alternate segment thm)<br>$\therefore \angle CBF = \angle EDF$ (proven)                                                                                     |

| Qns | Working                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
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| 7a  | <p>General term of <math>\left(1 - \frac{x}{5}\right)^8</math></p> $= \binom{8}{r} (1)^{8-r} \left(-\frac{x}{5}\right)^r$ $= \binom{8}{r} \left(-\frac{x}{5}\right)^r$ $(7+2x) \left(1 - \frac{x}{5}\right)^8$ $= (7+2x) \left( \dots + \binom{8}{3} \left(-\frac{x}{5}\right)^3 + \binom{8}{2} \left(-\frac{x}{5}\right)^2 + \dots \right)$ $x^3 \text{ term} = 7 \binom{8}{3} \left(-\frac{x}{5}\right)^3 + 2x \binom{8}{2} \left(-\frac{x}{5}\right)^2$ $= 7(56) \left(-\frac{x^3}{125}\right) + 2x(28) \left(\frac{x^2}{25}\right)$ $= -\frac{392}{125}x^3 + \frac{56}{25}x^3$ $\text{Coefficient of } x^3 = -\frac{112}{125}$ |
| b   | $\binom{n}{2} 3^{n-2} \left(\frac{1}{2}x\right)^2 = \binom{n}{2} 3^{n-2} \frac{1}{4}x^2$ $\binom{n}{3} 3^{n-3} \left(\frac{1}{2}x\right)^3 = \binom{n}{3} 3^{n-3} \frac{1}{8}x^3$ $\binom{n}{2} 3^{n-2} \frac{1}{4} = 3 \times \binom{n}{3} 3^{n-3} \frac{1}{8}$ $\binom{n}{2} 3^{n-2} \cdot 2 = \binom{n}{3} 3^{n-2}$ $\binom{n}{2} \cdot 2 = \binom{n}{3}$ $\frac{n(n-1)}{2} \cdot 2 = \frac{n(n-1)(n-2)}{2 \times 3}$ $6 = n - 2$ $n = 8$                                                                                                                                                                                       |

| Qns | Working                                                                                                                                                                                                                                                                                                                                                                                                                               |
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| 8a  | $\cos 2A = \frac{1}{3}$ $1 - 2\sin^2 A = \frac{1}{3}$ $\sin^2 A = \frac{1}{3}$ $\sin A = \frac{1}{\sqrt{3}} \quad \text{or} \quad -\frac{1}{\sqrt{3}} \quad (\text{rej})$ $\operatorname{cosec} A = \frac{1}{\sin A}$ $= 1 \div \frac{1}{\sqrt{3}}$ $= \sqrt{3}$                                                                                                                                                                      |
| b   | $\sin 105^\circ$ $= \sin(45^\circ + 60^\circ)$ $= \sin 45^\circ \cos 60^\circ + \cos 45^\circ \sin 60^\circ$ $= \frac{1}{\sqrt{2}} \cdot \frac{1}{2} + \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2}$ $= \frac{1 + \sqrt{3}}{\sqrt{2} \cdot 2}$ $\operatorname{cosec} A = \sqrt{2} \sin 105^\circ$ $= \sqrt{3} - \sqrt{2} \cdot \frac{1 + \sqrt{3}}{\sqrt{2} \cdot 2}$ $= \sqrt{3} - \frac{1 + \sqrt{3}}{2}$ $= \frac{\sqrt{3} - 1}{2}$ |

| Qns | Working |
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| 9a |  <p><math>a = -2 \cos \frac{1}{2} t</math></p>                                                                                                                                                                                                                                                                                                                                                                                       |
| b  | $v = \int -2 \cos \frac{1}{2} t \, dt$ $= \frac{-2 \sin \frac{1}{2} t}{\frac{1}{2}} + c$ $= -4 \sin \frac{1}{2} t + c$ <p>Given <math>t = 0</math> and <math>v = 3</math>, find <math>c</math></p> $3 = -4 \sin \frac{1}{2} (0) + c$ $c = 3$ $v = -4 \sin \frac{1}{2} t + 3$ <p>find <math>t</math> when <math>v = 0</math>,</p> $-4 \sin \frac{1}{2} t + 3 = 0$ $\sin \frac{1}{2} t = \frac{3}{4}$ <p>basic <math>\angle</math> of <math>\frac{1}{2} t = 0.84806</math></p> $t = 1.6961$ $t = 1.70 \text{ (3 s.f.)}$ |

| Qns | Working                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
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| c   | <p>basic <math>\angle</math> of <math>\frac{1}{2}t = 0.84806</math></p> <p>since <math>\sin \frac{1}{2}t</math> is +ve, <math>\frac{1}{2}t</math> is also in 2nd quad.</p> $\frac{1}{2}t = \pi - 0.84806$ $\frac{1}{2}t = 2.2935$ $t = 4.5870$ <p>Displacement</p> $= \int \left( -4 \sin \frac{1}{2}t + 3 \right) dt$ $= -4 \cdot \frac{-\cos \frac{1}{2}t}{\frac{1}{2}} + 3t + C_1$ $= 8 \cos \frac{1}{2}t + 3t + C_1$ <p>Distance travelled in first 5 min</p> $= \left  \left[ 8 \cos \frac{1}{2}t + 3t \right]_0^{1.6961} \right  + \left  \left[ 8 \cos \frac{1}{2}t + 3t \right]_{1.6961}^{4.5870} \right  + \left  \left[ 8 \cos \frac{1}{2}t + 3t \right]_{4.5870}^5 \right $ $=  10.379 - 8  +  8.4696 - 10.379  +  8.5908 - 8.4696 $ $= 2.3798 + 1.9100 + 0.1212$ $= 4.41 \text{ m}$ |

| Qns | Working                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
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| 10a | $y = 2x - 1 \text{----- (1)}$ $y = -x^{\frac{3}{2}} \text{----- (2)}$ $\text{fr (2), } y^2 = x^3 \text{---- (3)}$ $\text{sub (1) into (3), } (2x - 1)^2 = x^3$ $4x^2 - 4x + 1 = x^3$ $0 = x^3 - 4x^2 + 4x - 1$ $(x - 1) \text{ is a factor since } A(1,1) \text{ is intersection}$ $0 = (x - 1)(Ax^2 + Bx + C)$ $x^3 - 4x^2 + 4x - 1 = (x - 1)(Ax^2 + Bx + C)$ $\text{compare } x^3 : 1 = A$ $\text{compare constant: } -1 = -C$ $1 = C$ $\text{sub } x = 2, -1 = 2^2 + 2B + 1$ $-3 = B$ $(x - 1)(x^2 - 3x + 1) = 0$ $x - 1 = 0 \quad \text{or} \quad x^2 - 3x + 1 = 0$ $x = 1 \qquad \qquad \qquad x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$ $\qquad \qquad \qquad = 2.6180 \quad \text{or} \quad 0.381966$ $\text{sub } x = 0.381966, y = 2x - 1$ $y = 2(0.381966) - 1$ $= -0.236068$ $\therefore C = (0.38197, -0.23607) \text{ (shown)}$ |

| Qns | Working                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
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| b   | <p>sub <math>y = 0</math> to find <math>x</math> – coordinate of <math>B</math></p> $y = 2x - 1$ $0 = 2x - 1$ $x = 0.5$ <p>Let <math>A</math> be the shaded area above the <math>x</math>-axis<br/> Let <math>B</math> be the shaded area below the <math>x</math>-axis</p> $A = \left  \int_0^1 x^{\frac{3}{2}} dx \right  - \frac{1}{2}(0.5)(1)$ $= \left[ \frac{x^{\frac{5}{2}}}{\frac{5}{2}} \right]_0^1 - 0.25$ $= \frac{2}{5} - 0.25$ $= 0.15 \text{ units}^2$ $B = \int_0^{0.38197} -x^{\frac{3}{2}} dx + \frac{1}{2}(0.5 - 0.38197)(0.23607)$ $= \left[ -\frac{x^{\frac{5}{2}}}{\frac{5}{2}} \right]_0^{0.38197} + 0.013931$ $= 0.03606089 + 0.0139391$ $= 0.049999 \text{ units}^2$ <p>Total shaded area = <math>0.15 + 0.049999</math></p> $= 0.200 \text{ units}^2 (3 \text{ s.f.})$ |