

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\cdots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Solve the simultaneous equations

$$y - x = 4,$$

$$4x^2 - 6xy - 6y = 4.$$

2 Solve the equation $9 \sin x = -2 \sec x$ for $-\pi \leq x \leq 0$.

[4]



3 **(a)** Explain why the principal value of $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ cannot be $-\frac{\pi}{4}$.

[1]



(b) Prove that $\cot A + \tan A = \sec A \operatorname{cosec} A$.

[4]



4 Express $\frac{2x^3 + 4}{x(x+2)^2}$ in partial fractions.

[5]



5 The function f is given by $f(x) = \frac{x+a}{x^2+3}$.

(a) Find $f'(x)$.

[2]

It is given that f increases for $b < x < 1$.

(b) Find the values of a and b .

[4]



- 6 Find the set of values of the constant k for which the curve $y = (k - 6)x^2 - 8x + k$ does not intersect the x -axis and has a minimum point.

[6]



7 **(a)** Find $\frac{d}{dx}[(x-1)e^{-x}]$. [2]

(b) Hence find $\int xe^{-x} dx$. [4]



8 The radius of a circle, r cm, decreases at a rate of $\frac{2}{(t+1)^2}$ cm per minute.

(a) Given that the initial radius is 4 cm, find an expression for r in terms of t . [3]

(b) Find the rate of change of the area of the circle when the radius is 2.6 cm. [3]



- 9 (a) Find the value of the constant k such that the line $y = kx + 6$ is a tangent to the curve $2x^2 - xy = 3$.

[3]

- (b) Solve $(4 - 2\sqrt{2})x + \sqrt{2} = 3x\sqrt{2} - 1$, giving your answer in the form $a\sqrt{2} + b$, where a and b are rational numbers.

[4]



- 10** An isosceles triangle PQR has vertices $P(1, 0)$, $Q(0, 2)$ and $R(2, 3)$.
 $PQ = QR$.

(a) Find the area of the triangle PQR .

[2]

(b) If S is a point such that $PQRS$ is a kite and it also lies on the line $x - y = 5$, find the coordinates of S .

[5]

- 11 (a)** Solve the equation $2^x + 4^{x-1} = 24$.

[4]



(b) Solve the equation $\log_{\sqrt{3}}(x-1) = \log_3(x+5)$.

[4]



12 The gradient at any point on a curve y is given by $6x - \frac{1}{kx^3}$. The line $4x - 4y = 3$ is a normal to the curve at the point where $x = \frac{1}{2}$.

(a) Show that $k = 2$.

[2]

(b) Hence find the x -coordinates of the stationary points.

[2]



- (c) Find $\frac{d^2y}{dx^2}$ and explain whether the gradient $6x - \frac{1}{kx^3}$ has a turning point. [4]



13 The cost in thousands of dollars, C , of producing n hundred pairs of a certain type of running shoes is given by the formula $C = 1.2n^2 - 14.4n + 53.7$.

(a) Find the initial cost of production. [1]

(b) Express C in the form $a(x + h)^2 + k$ where a , h and k are constants to be determined. [3]

(c) Explain the meaning of the values of h and k in the formula as expressed in **part (b)**. [1]

(d) Find the maximum number of pairs of running shoes for which the cost will be [3]



50 thousand dollars.



- 14** The line $y = -x + 4$ is a tangent to a circle at the point A and the centre of the circle is at $B(-4, -2)$.

(a) Find coordinates of A .

[3]

(b) Find the radius and the equation of the circle.

[3]



- (c) Find the equation of another tangent to the circle which is parallel to $y = -x + 4$. [4]



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