

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\cdots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

- 1** The nutrient absorption rate, N , in mg/hour, of a plant depends on the diameter x , in mm, of its root hairs and is modelled by the function $N(x) = x^2 \ln\left(\frac{2}{x}\right)$, where $0 < x < 2$.

(a) Find $N'(x)$.

[3]

(b) Find the optimal root hair diameter that maximises nutrient absorption and show that the nutrient absorption is a maximum.

[5]



- 2 Hot coffee is heated to a temperature of 90°C . The temperature, $T^{\circ}\text{C}$, t minutes after removal from the heat source is given by $T = 22 + Ae^{kt}$, where A and k are constants.

(a) Explain why $A = 68$.

[1]

(b) After 3.5 minutes, the temperature of coffee is 79°C . Find k .

[3]



- (c) The coffee should only be served when it is between $55\text{ }^{\circ}\text{C}$ to $70\text{ }^{\circ}\text{C}$. Determine, with working, whether the coffee is suitable to be served 10 minutes after removal from the heat source.

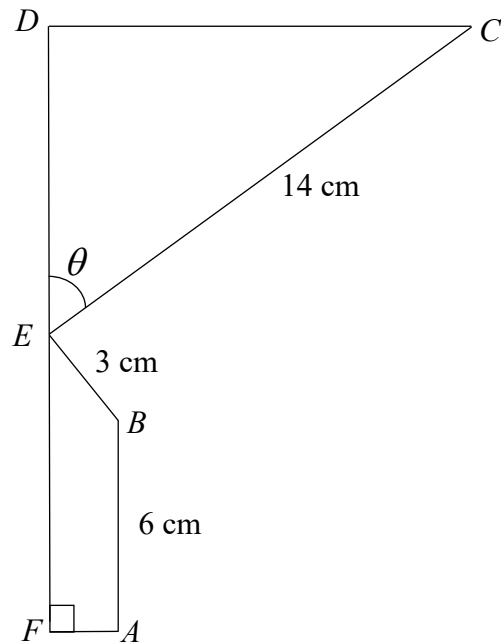
[2]

- (d) After a long time, T approaches a value. Calculate this value.

[1]



- 3 In the diagram, angle $CED = \theta$. EC is perpendicular to BE . The lengths of AB , BE and EC are 6 cm, 3 cm and 14 cm respectively. AB is parallel to FED and CD is parallel to AF .



- (a) Show that $FD = 14 \cos \theta + 3 \sin \theta + 6$.

[3]



- (b) Express FD in the form $R\cos(\theta - \alpha) + d$ where $R > 0$, $0^\circ < \alpha < 90^\circ$, and d is a constant.

[3]

- (c) Find the value of θ when $FD = 12$ cm.

[2]

- (d) Determine if it is possible for FD to be 15 cm.

[2]



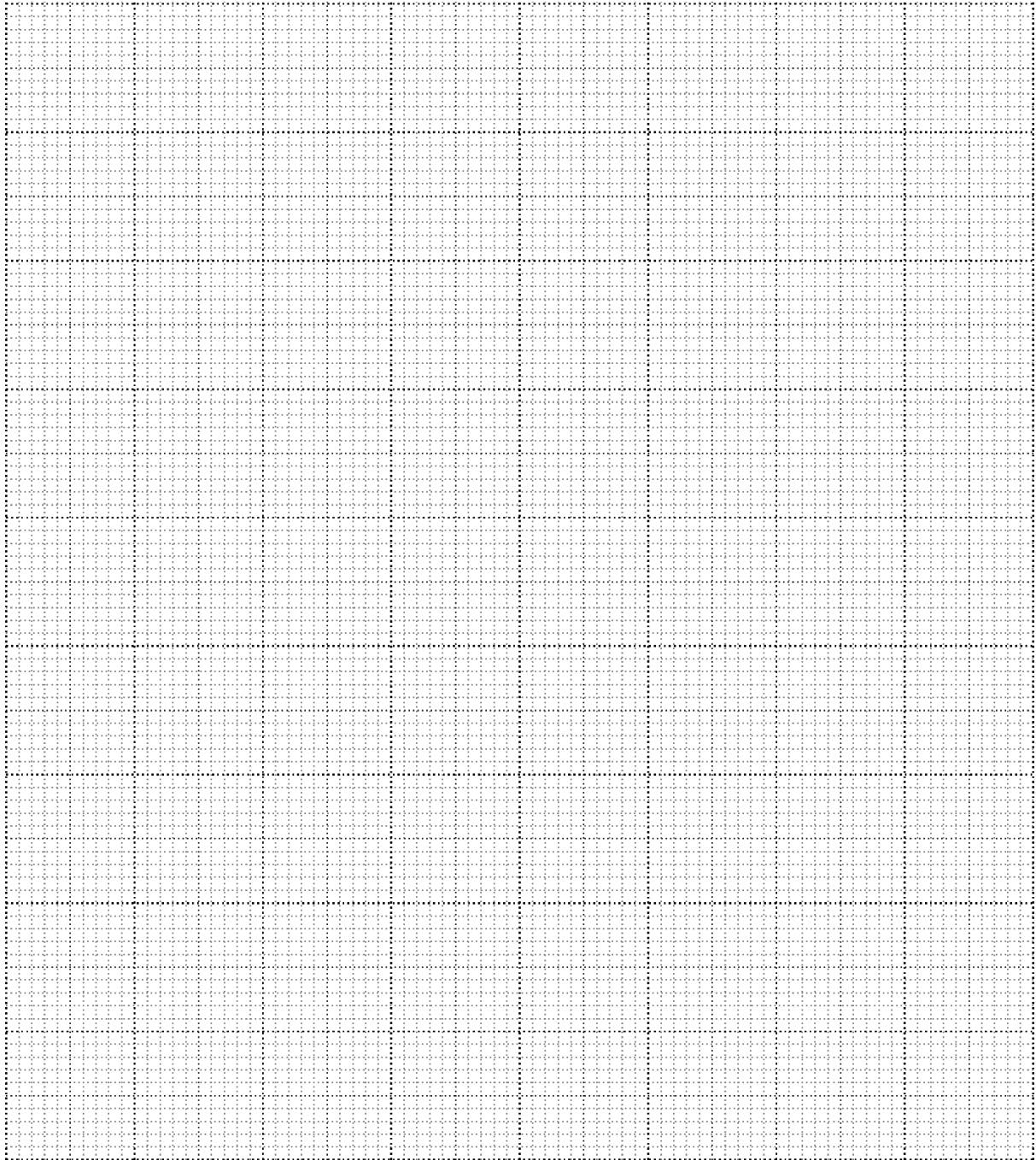
- 4 In a biology experiment, a certain type of bacteria multiplies rapidly. The number of bacteria, N , after t hours is modelled by the formula $N = a10^{kt}$, where a and k are constants. The table below shows corresponding values of t and N .

t	2	4	6	8
N	900	2700	8100	24 300

- (a) On the grid on the next page, plot $\lg N$ against t and draw a straight line graph to illustrate the information.

[4]





(b) Use the graph to estimate the value of each of the constants a and k .

[2]

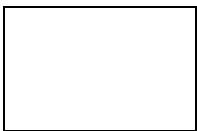


- 5 (a) The cubic polynomial $f(x)$ is such that the coefficient of x^3 is -2 and the roots of $f(x) = 0$ are -1 , 2 and $\frac{3}{2}$. Find the expression of $f(x)$ in descending powers of x . [3]

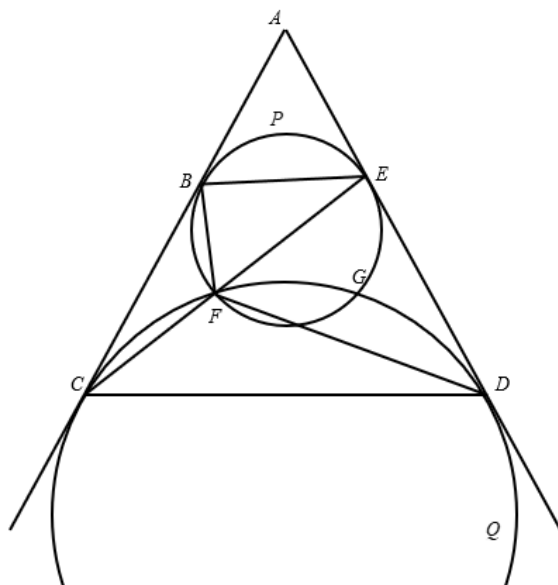
- (b) Find the quotient when $x^3 - 3x^2 - 2x + 5$ is divided by $x^2 - 1$. [2]

- (c) The expression $hx^3 - 7x^2 + kx - 2$, where h and k are constants, has a factor $3x^2 - 5x - 2$. Find the value of h and k . [5]





- 6 The diagram shows two circles P and Q which intersect each other at F and G . The line ABC is tangent to P and Q at B and C respectively. The line AED is tangent to P and Q at E and D respectively. EFC is a straight line.



- (a) Prove that triangle ABE is similar to triangle ACD .

[4]



(b) Explain why a circle can be drawn to pass through $BEDC$.

[3]

(c) Prove that $\text{angle } CBF = \text{angle } EDF$.

[3]



- 7 **(a)** Find the coefficient of x^3 in the expansion of $(7+2x)\left(1-\frac{x}{5}\right)^8$. [4]



- (b) In the expansion of $\left(3 + \frac{x}{2}\right)^n$, the coefficient of x^2 is three times the coefficient of x^3 .
Find n . [4]



8 Do not use a calculator in this question.

(a) Find the exact value of $\operatorname{cosec} A$ when $\cos 2A = \frac{1}{3}$ and A is acute.

[4]



(b) Hence, find the exact value of $\operatorname{cosec} A - \sqrt{2} \sin 105^\circ$.

[5]



- 9** A particle moves in a straight line so that its acceleration, $a \text{ m/s}^2$, is given by $a = -2 \cos \frac{1}{2}t$, where t is the time in seconds after leaving a fixed point O .

(a) Sketch the acceleration-time graph of the particle for $0 \leq t \leq 2\pi$. [2]

(b) The initial velocity of the particle is 3 m/s. Find the earliest time when the particle first comes to rest. [4]

Continuation of working space for question **9(b)**.

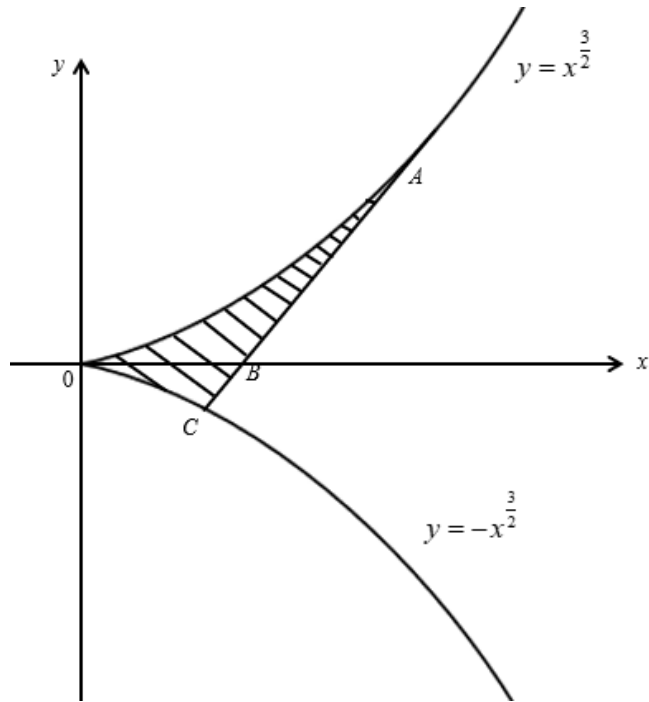


(c) Find the total distance travelled in the first 5 seconds.

[5]



- 10** The diagram shows the graphs of $y = x^{\frac{3}{2}}$ and $y = -x^{\frac{3}{2}}$. The line $y = 2x - 1$ intersects the graph of $y = x^{\frac{3}{2}}$ at $A(1, 1)$, the x -axis at B , and the graph of $y = -x^{\frac{3}{2}}$ at C .



- (a) Show that the coordinates of C are $(0.38197, -0.23607)$, correct to 5 significant figures.

[5]



Continuation of working space for question **10(a)**.

(b) Find the area of the shaded region.

[6]



Continuation of working space for question **10(b)**.

End of Paper

