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|-------|--|---------------|--|--------|--|
| Name: |  | Index Number: |  | Class: |  |
|-------|--|---------------|--|--------|--|

on  
**Secondary 4 (O-Level Programme)**

**A**

**Additional Mathematics**

**4049/02**

Paper 2

**1 September 2025**

**2 hours 15 minutes**

Booklet A

Additional materials: Question Booklets B and C

**READ THESE INSTRUCTIONS FIRST**

Write your name, index number and class on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions in the space provided.

If working is needed for any question, it must be shown with the answer.

Omission of essential working will result in loss of marks.

Calculators should be used where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer answer to **three significant figures**.

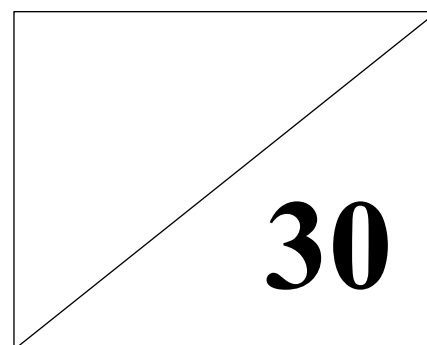
Give answers in **degrees to one decimal place**.

For  $\pi$ , use either your calculator value or 3.142, unless the question requires the answer in terms of  $\pi$ .

The number of marks is given in brackets [ ] at the end of each question or part question.

The total mark for this paper is **90**.

*For examiners' use*



## ***Mathematical Formulae***

### **1. ALGEBRA**

#### *Quadratic Equation*

For the quadratic equation  $ax^2 + bx + c = 0$ ,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

#### *Binomial Expansion*

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where  $n$  is a positive integer and  $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

### **2. TRIGONOMETRY**

#### *Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

#### *Formulae for $\Delta ABC$*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

- 1** Jane bought an electric car in January 2025. After purchase, the value of the car,  $\$V$ , diminishes over time and can be modelled by  $V = 38000e^{-kt}$ , where  $k$  is a positive constant and  $t$  is time measured in years. The value of the car is expected to be  $\$29000$  after 3 years of driving. Jane intends to sell her car when its value drops to half of its original value.

Showing clear mathematical calculations, find the year that Jane is likely to sell her electric car. [5]

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- 2**     **(i)**     Given that the curve  $y = ax^2 + 2bx - 1$  lies entirely below the  $x$ -axis, determine the conditions that must be applied to the constants  $a$  and  $b$ . [2]

- (ii)**   If  $a$  and  $b$  are both integers, state an example of the values of  $a$  and  $b$  which satisfy the conditions found in **(i)**. [2]

- 
- 3**     **(a)**     The variables  $x$  and  $y$  are defined such that  $\log_2 x - 3 \log_4 y = \log_9 3$ .

- (i)**     Give a reason why  $x$  and  $y$  must be positive numbers. [1]

**(ii)** Express  $x$  in terms of  $y$ .

[5]

**(b)** Solve the equation  $5^{x+1} + 2(5^{-x}) = 11$ , leaving non-exact value(s) of  $x$  in the form  $\log_a b$ . [4]

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- 4      (a)    (i)    The polynomials  $P(x) = ax^3 + x^2 + bx + 10$  and  $Q(x) = x^3 + ax^2 - 5x + b$  leave the same remainder when divided by  $x + 2$ .

Show that  $4a + b = 4$ .

[2]

- (ii) If  $P'(x)$  has a factor of  $3x - 2$ , find the values of  $a$  and  $b$ .

[4]

- (b) Solve the equation  $2x^3 + 4x^2 - 5x - 7 = 0$ , expressing non-exact solutions in the form  $a \pm \frac{1}{2}\sqrt{b}$  where  $a$  and  $b$  are constants to be determined.

[5]

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on  
**Secondary 4 (O-Level Programme)**

**B**

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**Additional Mathematics**

**4049/02**

Paper 2

**1 September 2025**

Booklet B

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**READ THESE INSTRUCTIONS FIRST**

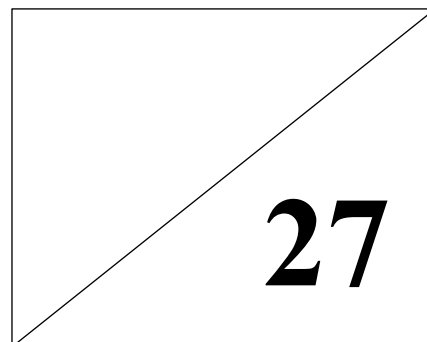
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*For examiners' use*



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- 5** The depth of water,  $h$  metres, in a shallow port on a particular day is modelled by the formula  $h = 7 + 2 \sin\left(\frac{\pi}{6}t\right)$ , where  $t$  is the number of hours after midnight.

**(a)** State the period of  $h$ . [1]

**(b)** Use the model to predict the time when the depth of water is at its lowest. [2]

A supply boat docks at the port at 5.30 am and it takes the workers 3 hours to unload its cargo immediately after it docks. To leave the port safely, the supply boat requires the depth of water to be at least 5.6 metres.

**(c)** Determine the earliest time, to the nearest minute, that the supply boat can leave the port. [4]

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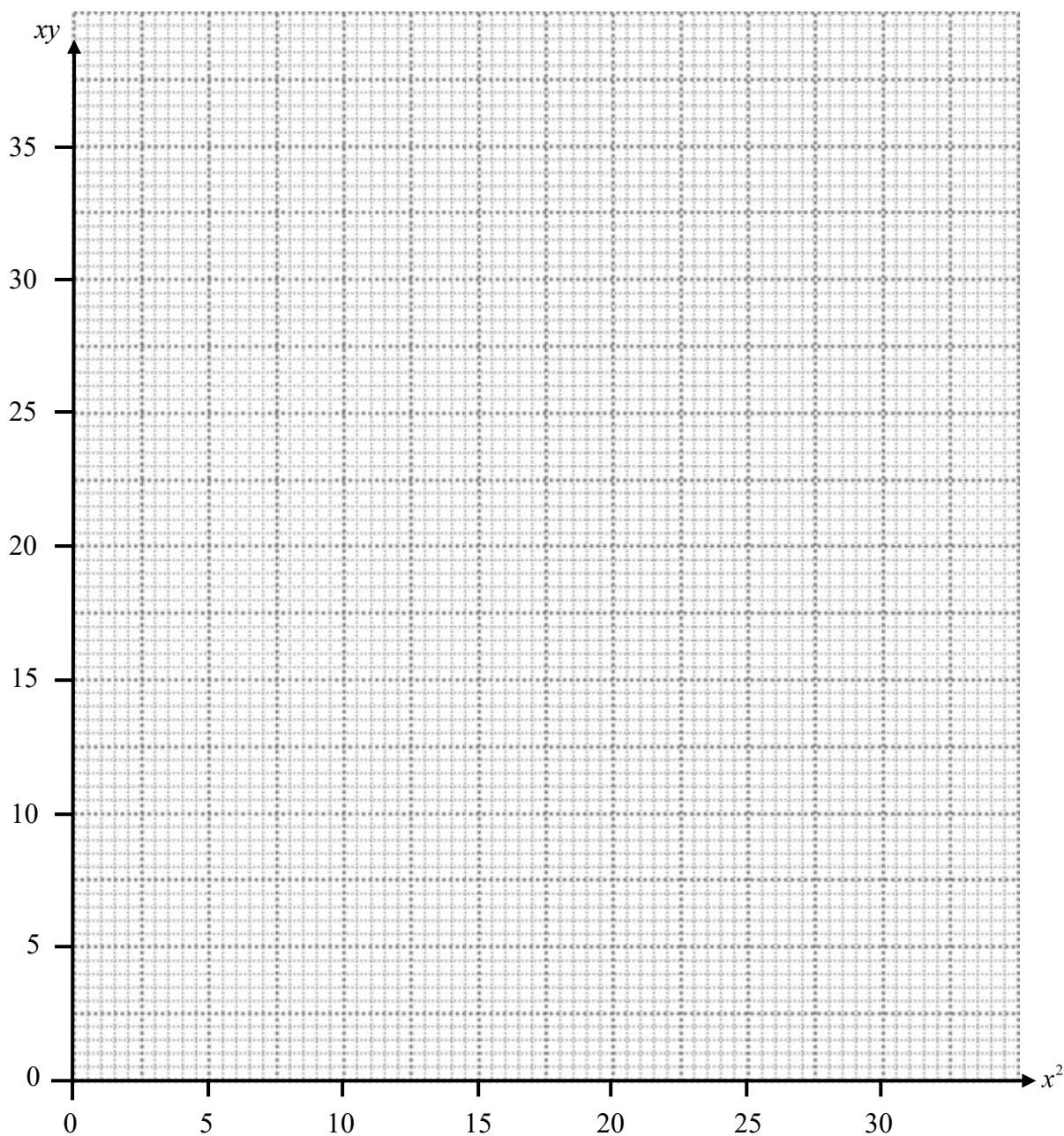
- 6 The variables  $x$  and  $y$  are known to be related by the formula  $y = px + \frac{q}{x}$  where  $p$  and  $q$  are constants.

(a) The table below shows some experimental values of  $x$  and  $y$ .

|     |      |      |      |      |      |
|-----|------|------|------|------|------|
| $x$ | 1    | 2    | 3    | 4    | 5    |
| $y$ | 8.55 | 6.20 | 5.32 | 5.72 | 6.00 |

(i) On the grid below, plot  $xy$  against  $x^2$  and draw a straight line graph.

[2]



**(ii)** Use your graph to estimate

**(a)** the value of  $p$  and of  $q$ .

[3]

**(b)** the positive value of  $x$  that satisfies the equation  $px + \frac{q}{x} = \frac{20}{x}$ .

[2]

**(b)** Explain how another straight line graph can be obtained from  $y = px + \frac{q}{x}$  by plotting  $\frac{x}{y}$  against  $\frac{1}{xy}$ , expressing clearly the gradient and vertical intercept in terms of  $p$  and/or  $q$ .

You need not draw this straight line graph.

[3]

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7 The equation of a curve is  $y = \frac{\ln 2x}{x}$ .

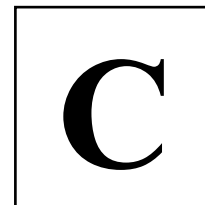
(i) Find an expression for  $\frac{d^2y}{dx^2}$  and show that  $x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + \frac{1}{x^2} = 0$ . [5]

- (ii) Find the **exact** coordinates of the stationary point of the curve and use the second derivative test to determine if the stationary point is a maximum or minimum. [5]

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on  
**Secondary 4 (O-Level Programme)**



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**Additional Mathematics**

**4049/02**

Paper 2

**1 September 2025**

Booklet C

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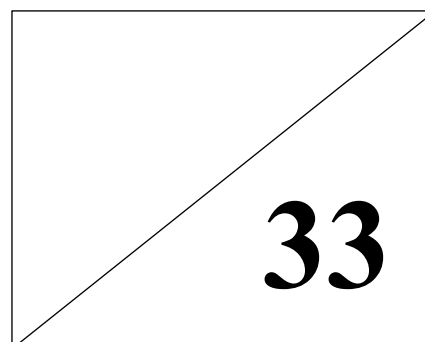
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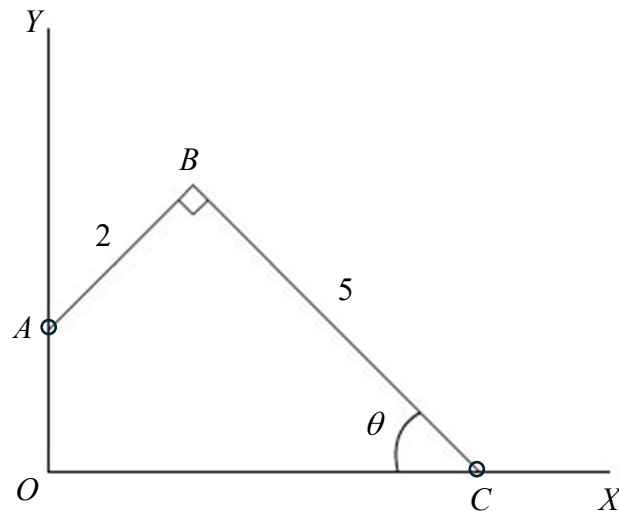
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- 8 The diagram shows two rods  $AB$  and  $BC$  of length 2 m and 5 m respectively, joined together at  $B$  such that angle  $ABC = 90^\circ$ . Small rings are attached to  $A$  and  $C$  so that  $A$  can move along the vertical pole  $OY$  and  $C$  can move along the horizontal pole  $OX$ . The rod  $BC$  makes an angle  $\theta$  with  $OX$ .



- (i) Show that  $OA = 5 \sin \theta - 2 \cos \theta$ . [2]

- (ii) Express  $OA$  in the form  $R \sin(\theta - \alpha)$  where  $R > 0$  and  $0^\circ < \alpha < 90^\circ$ . [3]

- (iii) Given that  $A$  can be no higher than 3.5 m above  $O$ , calculate the greatest possible value of  $\theta$ . [2]

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9 A particle moves in a straight line with a fixed point  $O$ . The velocity of the particle,  $v$  m/s, is given by  $v = \frac{25}{2t+1} + 10$  where  $t$  is time in seconds after passing  $O$ .

- (a) Explain why the particle never change its direction of motion and will eventually travel at a speed close to 10 m/s. [2]

- (b) Find the acceleration of the particle at  $t = 2$ . [2]

(c) Calculate the distance travelled by the particle in the fifth second of its motion.

[4]

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**10**  $A(1, 7)$  and  $B(9, 7)$  lie on a circle  $C_1$  with centre  $P$ . The line  $4y + 3x = 31$  is a normal to the circle and passes through  $A$ .

(i) Use a geometrical property of circle to explain why the  $x$ -coordinate of  $P$  is 5.

[2]

(ii) Find the equation of the circle  $C_1$ .

[3]

(iii) The tangent to the circle at  $A$  intersects the  $y$ -axis at  $D$ .

Find the coordinates of  $D$ .

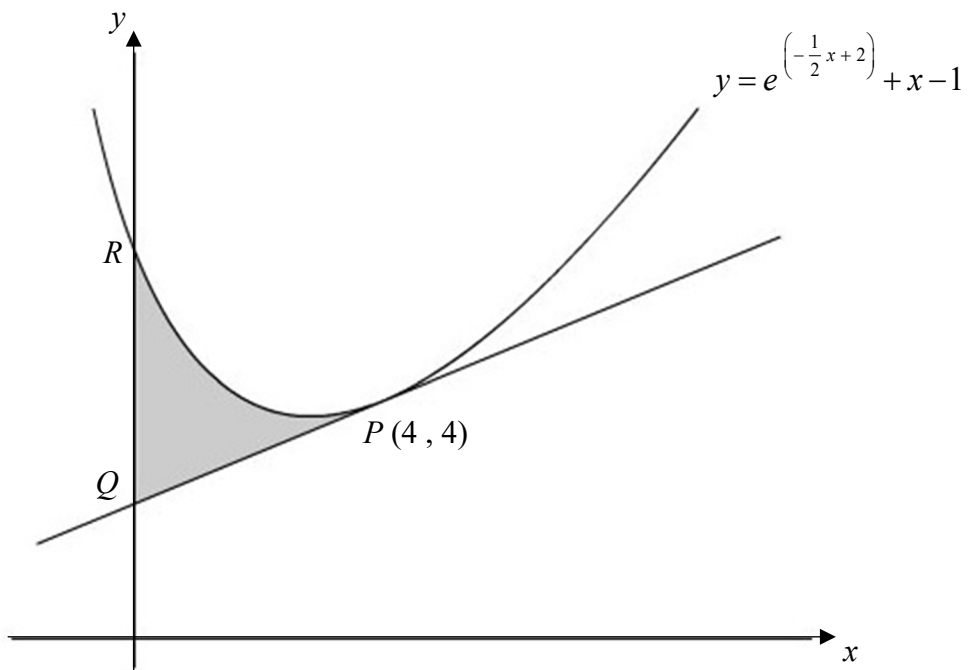
[3]

A larger circle  $C_2$  has the same centre  $P$  and radius three times that of  $C_1$ .  $T$  is a point on  $C_2$  such that  $P$ ,  $A$  and  $T$  are collinear and the  $x$ -coordinate of  $T$  is negative.

(iv) Determine the coordinates of  $T$ .

[2]

- 11 The diagram shows part of the curve  $y = e^{\left(-\frac{1}{2}x+2\right)} + x - 1$  that intersects the  $y$ -axis at  $R$ . The tangent to the curve at  $P(4, 4)$  intersects the  $y$ -axis at  $Q$ .



Show that the area of the shaded region can be expressed as  $ae^2 + b$  where  $a$  and  $b$  are constants to be determined. [8]

(Continuation of working space for question 11.)

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**END OF PAPER**