

Name:		Index Number:		Class:	
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Preliminary Examination Secondary 4 (O-Level Programme)

Mathematics

4052/01

Paper 1

25 August 2025

2 hours 15 minutes

Additional Materials: Booklets A and B

Candidates answer in the space provided on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions in the space provided.

If working is needed for any question, it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to **three significant figures**. Give answers in **degrees to one decimal place**.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [] at the end of each question or part question.

The total of the marks for this paper is **90**.

For Examiner's Use
90

MATHEMATICAL FORMULAE

Compound interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of a triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

1 Simplify $\frac{5}{2-x} + \frac{4x-8}{(x-2)^2}$.

$$\begin{aligned} & \frac{5}{2-x} + \frac{4x-8}{(x-2)^2} \\ &= -\frac{5}{x-2} + \frac{4(x-2)}{(x-2)^2} \quad \text{or} \quad = \frac{5}{2-x} - \frac{4}{2-x} \\ &= -\frac{1}{x-2} \quad \text{or} \quad = \frac{1}{2-x} \end{aligned}$$

Answer _____ [3]

2 Rearrange the formula to make p the subject.

$$pr = \frac{6p^2 - 1}{p}$$

$$pr = \frac{6p^2 - 1}{p}$$

$$p^2(6-r) = 1$$

$$p = \pm \sqrt{\frac{1}{6-r}}$$

Answer _____ [2]

3 Factorise the following expressions completely.

(a) $5m + 2n - mn - 10$
 $5m + 2n - mn - 10$
 $= 5(m - 2) + n(2 - m)$
 $= (5 - n)(m - 2)$

or

$$= (n - 5)(2 - m)$$

Answer _____ [2]

(b) $x^2 + 4xy - 9 + 4y^2$

$$\begin{aligned} & x^2 + 4xy + 4y^2 - 9 \\ &= (x + 2y)^2 - 9 \\ &= (x + 2y + 3)(x + 2y - 3) \end{aligned}$$

Answer _____ [2]

- 4 The product of two numbers, x and y , is 6300 and their highest common factor is 6. Given $6 < x < y$, find the value of x and the value of y .

$$6300 = 2^2 \times 3^2 \times 5^2 \times 7$$

$$x = 2 \times 3 \times 7 = 42$$

$$y = 2 \times 3 \times 5^2 = 150$$

Answer $x =$ _____

$y =$ _____ [2]

- 5 Simplify $\left(\frac{49x^4}{y^{16}}\right)^{-\frac{3}{2}}$, leaving your answer in positive indices.

$$\left(\frac{49x^4}{y^{16}}\right)^{-\frac{3}{2}} = \frac{y^{24}}{343x^6}$$

Answer _____ [2]

- 6 (a) Express $-x^2 - 8x - 19$ in the form of $-(x + p)^2 + q$.

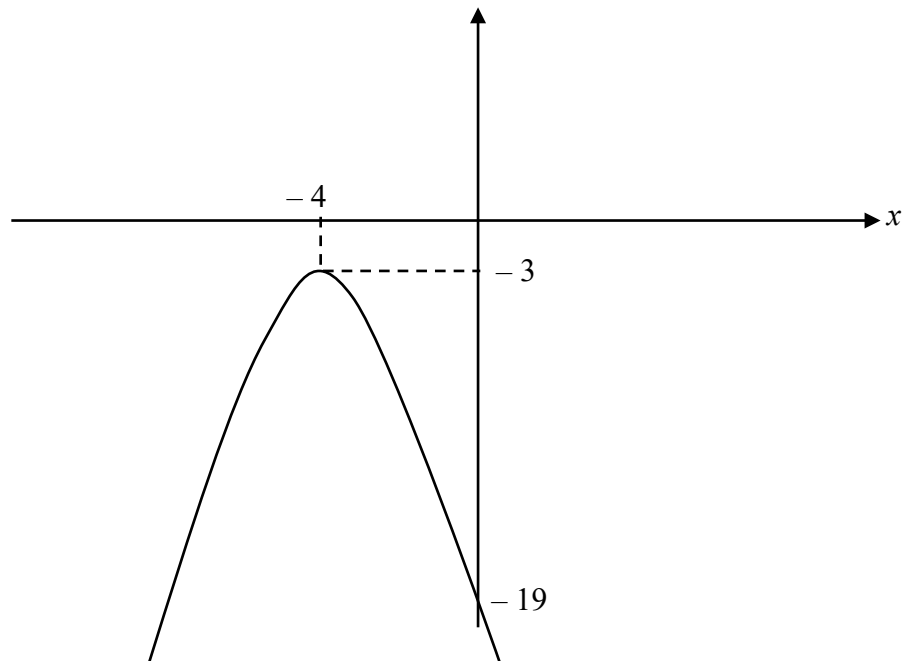
$$\begin{aligned} & -x^2 - 8x - 19 \\ & = -(x + 4)^2 - 3 \end{aligned}$$

Answer _____ [2]

- (b) Hence, on the axes below, sketch the graph of $y = -x^2 - 8x - 19$.

Answer

[2]



7 Andy places an order for some polo shirts that come in three sizes S , M and L .

The ratio of the number of polo shirts $S : M = 3 : 8$.

The ratio of the number of polo shirts $S : L = 5 : 2$.

There are 306 more polo shirts in size M than L .

Find the total number of polo shirts ordered.

$$S : M \qquad S : L$$

$$3 : 8 \qquad 5 : 2$$

$$15 : 40 \qquad 15 : 6$$

$$S : M : L$$

$$15 : 40 : 6$$

$$\begin{aligned} \text{Total} &= \left(\frac{306}{40-6} \right) (61) \\ &= 549 \end{aligned}$$

Answer _____ [2]

8 Solve $2^p \times 6 = 96$.

$$2^p \times 6 = 96$$

$$2^p = 16$$

$$p = 4$$

Answer $p =$ _____ [2]

9 Solve the inequality $x - 6 \leq 3x - 17 < 32 - 4x$.

Show your solution on the number line below.

$$x - 6 \leq 3x - 17 < 32 - 4x$$

$$3x - 17 \geq x - 6$$

$$3x - 17 < 32 - 4x$$

$$x \geq 5\frac{1}{2}$$

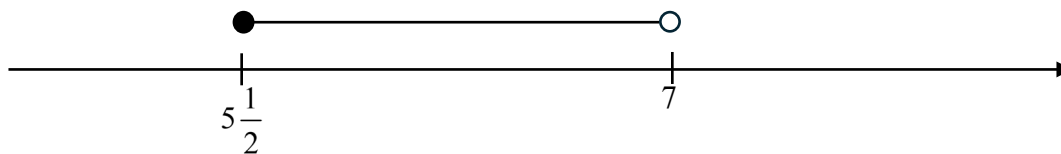
$$x < 7$$

$$5\frac{1}{2} \leq x < 7$$

Answer _____ [2]

Answer

[1]



10 The cash price of a refrigerator is \$2499.

The hire-purchase price of the refrigerator is \$2999.

The hire-purchase price is a deposit of 25% of the cash price plus 24 equal monthly payments.

Calculate one monthly payment.

$$\text{Remaining amount} = 2999 - (2499)(0.25) = 2374.25$$

$$\text{Monthly Instalment} = \frac{2374.25}{24} = 98.93$$

Answer \$ _____ [2]

- 11** Michael deposited some money into bank A which pays a simple interest rate of 3% per annum for 18 months.

If he had deposited the same amount of money with bank B, which pays a simple interest rate of 2.5% per annum for 24 months, he would receive \$500 more in interest.

- (a)** Find the amount of money he deposited.

$$\frac{P(2.5)(2)}{100} - \frac{P(3)(1.5)}{100} = 500$$
$$P = 100000$$

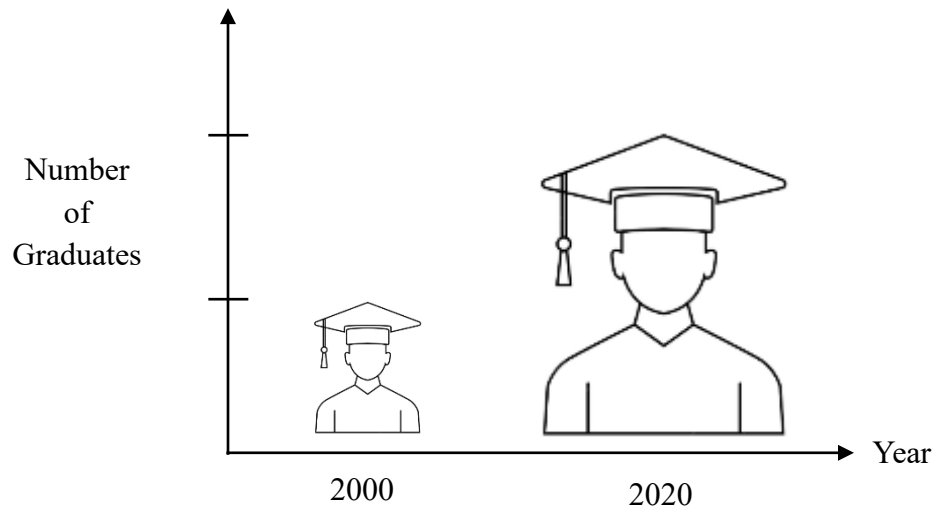
Answer \$ _____ [2]

- (b)** Find the amount of interest Michael would receive if he were to deposit his money into bank C which pays a compound interest of 1.9% per annum compounded every 6 months for 24 months.

$$A = 100000 \left(1 + \frac{1.9}{200} \right)^4$$
$$= 103854.49$$
$$I = 3854.49$$

Answer \$ _____ [2]

- 12 The number of university graduates for the year 2000 and year 2020 are represented by the chart below. Statistics has shown that the number of university graduates in year 2020 is doubled the number of university graduates in year 2000.



Explain how the above chart may be misleading and suggest a possible way to improve the chart.

[2]

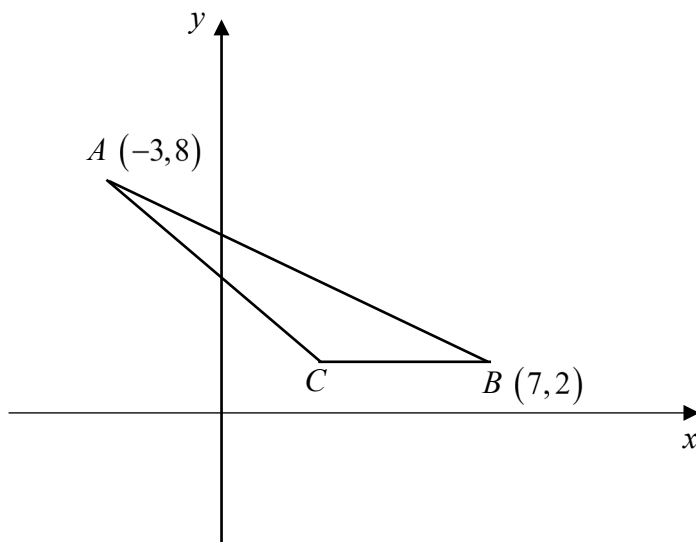
The **area** of the picture for year 2020 is more than twice the **area** of the picture for year 2000, which might mislead people into thinking that the number of graduates in 2020 is more than twice that of year 2000.

1. Replace with bar graphs of equal width and the height of bar of year 2020 twice the height of bar of year 2000.
2. Replace year 2020 picture with two identical 2000 pictures, and stack the two pictures on top of each other.

- 13 The vertices of A and B are $(-3,8)$ and $(7,2)$.

The line BC is parallel to the x – axis.

The area of triangle ABC is 12 square units.



- (a) Find the coordinates of C .

$$\frac{1}{2}(b)(6) = 12$$

$$b = 4$$

$$C(3,2)$$

Answer _____ [2]

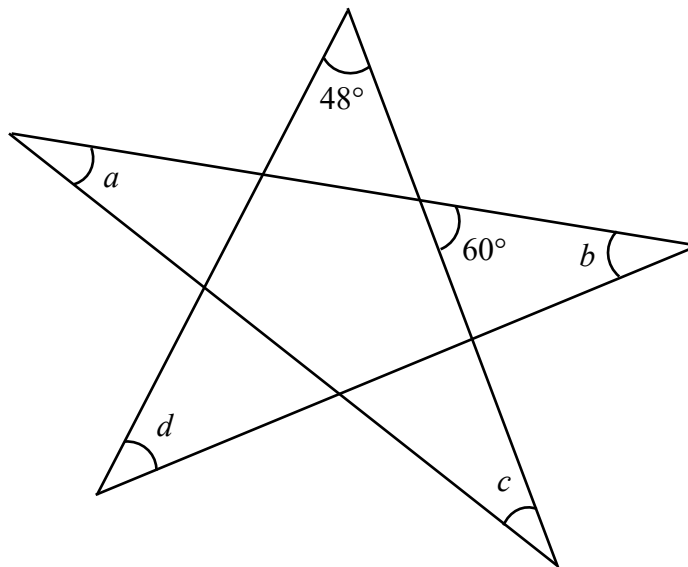
- (b) Find the exact value of $\cos \hat{ACB}$.

$$AC = \sqrt{(6^2) + 6^2} = 6\sqrt{2}$$

$$\cos \hat{ACB} = -\frac{6}{6\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

Answer _____ [2]

14 Find $\angle a + \angle b + \angle c + \angle d$.



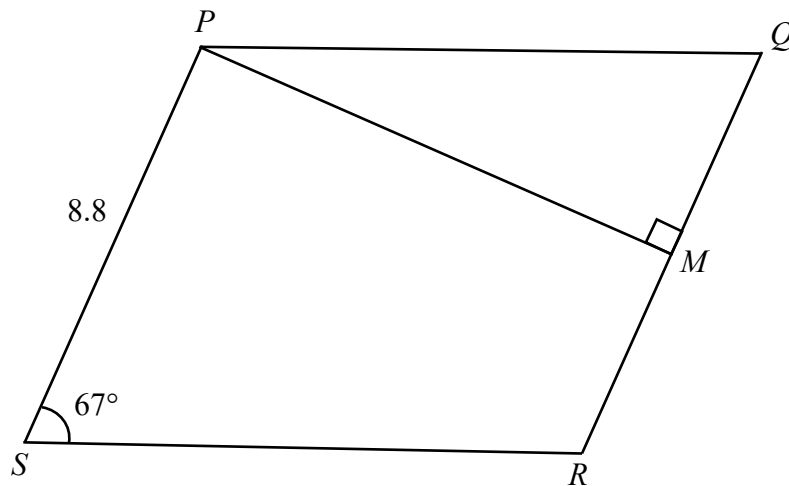
$$\angle a + \angle c = 60^\circ \text{ (exterior angle)}$$

$$\begin{aligned}\angle b + \angle d &= 180 - 60 - 48 \\ &= 72^\circ \text{ (exterior angle)}\end{aligned}$$

$$\begin{aligned}\angle a + \angle b + \angle c + \angle d &= 60 + 72 \\ &= 132^\circ\end{aligned}$$

Answer _____ $^\circ$ [2]

- 15** $PQRS$ is a parallelogram. PM is perpendicular to QR and M is the midpoint of QR .
 $PS = 8.8\text{cm}$ and angle $PSR = 67^\circ$.
 Find the area of $PQRS$.



$$\tan 67 = \frac{PM}{4.4}$$

$$PM = 4.4 \tan 67$$

$$\begin{aligned} \text{Area} &= (4.4 \tan 67) 8.8 \\ &= 91.2 \end{aligned}$$

Answer cm^2 [3]

16 The first four terms of a sequence are 1, 5, 9 and 13.

(a) Find an expression, in terms of n , for the n th term of the sequence.

$$4n - 3$$

Answer _____ [1]

(b) Find an expression, in terms of n , for the $(n + 1)$ th term of the sequence.

$$4(n + 1) - 3$$

$$= 4n + 1$$

Answer _____ [1]

(c) Explain why the difference between the squares of two consecutive terms is always a multiple of 8.

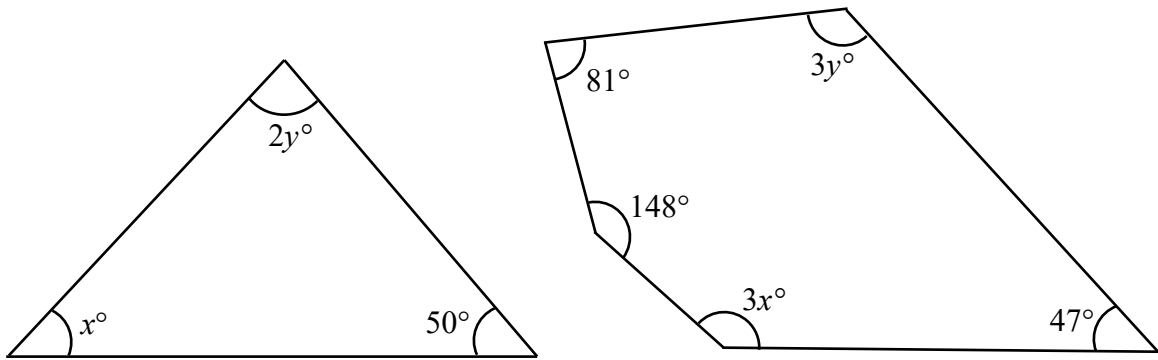
$$(4n + 1)^2 - (4n - 3)^2$$

$$= 16n^2 + 8n + 1 - (16n^2 - 24n + 9)$$

$$= 32n - 8$$

$$= 8(4n - 1)$$

$(4n - 1)$ is an integer and since 8 is a factor, the difference is always multiple of 8.



- (a) Using the information shown in the diagrams, write down and simplify two equations in x and y .

Answer

[3]

$$x + 2y = 130$$

$$3x + 3y = 540 - 81 - 148 - 47$$

$$x + y = 88$$

- (b) Solve these equations to find the value of x and of y .

$$x + 2y = 130 \quad -(1)$$

$$x + y = 88 \quad -(2)$$

$$(1) - (2), y = 42$$

$$x = 46$$

Answer $x =$ _____

$y =$ _____ [2]

- 18 (a) In a company of 120 workers, the ratio of female worker to male workers is 2 : 3.

	Female	Male
Full time	p	q
Part time	9	r

20% of the workers work part time.

Find the value of p , of q and of r .

$$\begin{aligned} r &= (120)(0.2) - 9 \\ &= 15 \end{aligned}$$

$$\begin{aligned} p &= (120)\left(\frac{2}{5}\right) - 9 \\ &= 39 \end{aligned}$$

$$\begin{aligned} q &= 120 - 9 - 39 - 15 \\ &= 57 \end{aligned}$$

Answer $p =$ _____

$q =$ _____

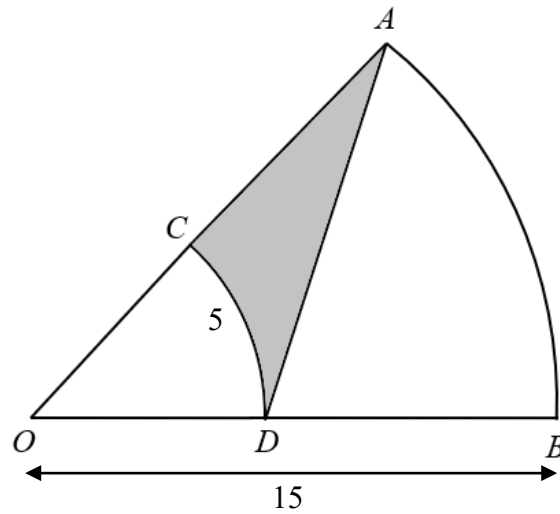
$r =$ _____ [2]

- (b) The annual salary of a particular male worker is \$85000 in 2023. His salary in 2023 was 12% higher than in 2021. Find his annual salary in 2021.

$$\begin{aligned} &\frac{85000}{1.12} \\ &= 75892.86 \end{aligned}$$

Answer \$ _____ [1]

- 19 The diagram shows two sectors OAB and OCD with centre at O . OB is 15 cm and CD has an arc length of 5 cm. The ratio of area of sector OCD : area of $ABDC$ is 16 : 65.



- (a) Find angle COD in radian.

Mtd 1:

$$\begin{aligned}\frac{A_{OCD}}{A_{OAB}} &= \frac{16}{81} \\ \frac{OD}{OB} &= \sqrt{\frac{16}{81}} \\ \frac{OD}{15} &= \frac{4}{9} \\ OD &= 6\frac{2}{3} \\ COD &= \frac{5}{6\frac{2}{3}} \\ &= \frac{3}{4} \text{ rad}\end{aligned}$$

Mtd 2:

$$\begin{aligned}\frac{A_{OCD}}{A_{OAB}} &= \frac{16}{81} \\ \frac{CD}{AB} &= \sqrt{\frac{16}{81}} \\ \frac{5}{AB} &= \frac{4}{9} \\ AB &= 11.25 \\ COD &= \frac{11.25}{15} \\ &= \frac{3}{4} \text{ rad}\end{aligned}$$

Mtd 3:

Let $\angle COD = \theta$ and $OD = r$.

$$r = \frac{5}{\theta}$$

$$\frac{\frac{1}{2} \left(\frac{5}{\theta} \right)^2 \theta}{\frac{1}{2} (15)^2 \theta - \frac{1}{2} \left(\frac{5}{\theta} \right)^2 \theta} = \frac{16}{65}$$

$$\frac{\frac{25}{2\theta^2}}{\frac{225}{2} - \frac{25}{2\theta^2}} = \frac{16}{65}$$

$$\frac{25}{225\theta^2 - 25} = \frac{16}{65}$$

$$\theta^2 = \frac{9}{16}$$

$$\theta = \frac{3}{4} \text{ rad}$$

Mtd 4:

Let $\angle COD = \theta$ and $OD = r$.

$$r = \frac{5}{\theta}$$

$$\text{Area of sector } OAB = \frac{1}{2} (15)^2 \theta = \frac{225}{2} \theta$$

$$\begin{aligned} \text{Area of sector } OAB &= \frac{16}{81} \times \frac{225}{2} \theta \\ &= \frac{200}{9} \theta \end{aligned}$$

$$\frac{1}{2} \left(\frac{5}{\theta} \right)^2 \theta = \frac{200}{9} \theta$$

$$\theta^2 = \frac{9}{16}$$

$$\theta = \frac{3}{4} \text{ rad}$$

Answer _____ rad [3]

- (b) Find the area of the shaded region.

Mtd 1:

$$\begin{aligned} A &= \frac{1}{2}(15)\left(6\frac{2}{3}\right)\sin\frac{3}{4} - \frac{1}{2}\left(6\frac{2}{3}\right)^2\left(\frac{3}{4}\right) \\ &= 34.082 - 16.667 \\ &= 17.4 \end{aligned}$$

Mtd 2:

Let h = perpendicular height of A to OB

$$\begin{aligned} h &= 15 \sin \frac{3}{4} \\ &= 10.22 \end{aligned}$$

$$\begin{aligned} A &= \frac{1}{2}\left(6\frac{2}{3}\right)(10.22) - \frac{1}{2}\left(6\frac{2}{3}\right)^2\left(\frac{3}{4}\right) \\ &= 17.4 \end{aligned}$$

Answercm² [3]

- 20 In this table, p is inversely proportional to q^2 .

p	0.08	0.5	b
q	a	4	2

Calculate the value of a and of b .

$$0.5 = \frac{k}{4^2}$$

$$k = 8$$

$$a = 10$$

$$b = 2$$

Answer $a =$ _____

$b =$ _____ [3]

- 21 (a)** A river with an actual length of 1 km is represented on a map with a length of 8 cm.
Find the map scale in the form 1 : n .

8 : 100,000

1 : 12500

Answer _____ [1]

- (b)** A forest is represented by an area of 200 cm² on the map. Calculate the actual area of forest in square metres, giving your answer in standard form.

1 : 125

1 : 15625

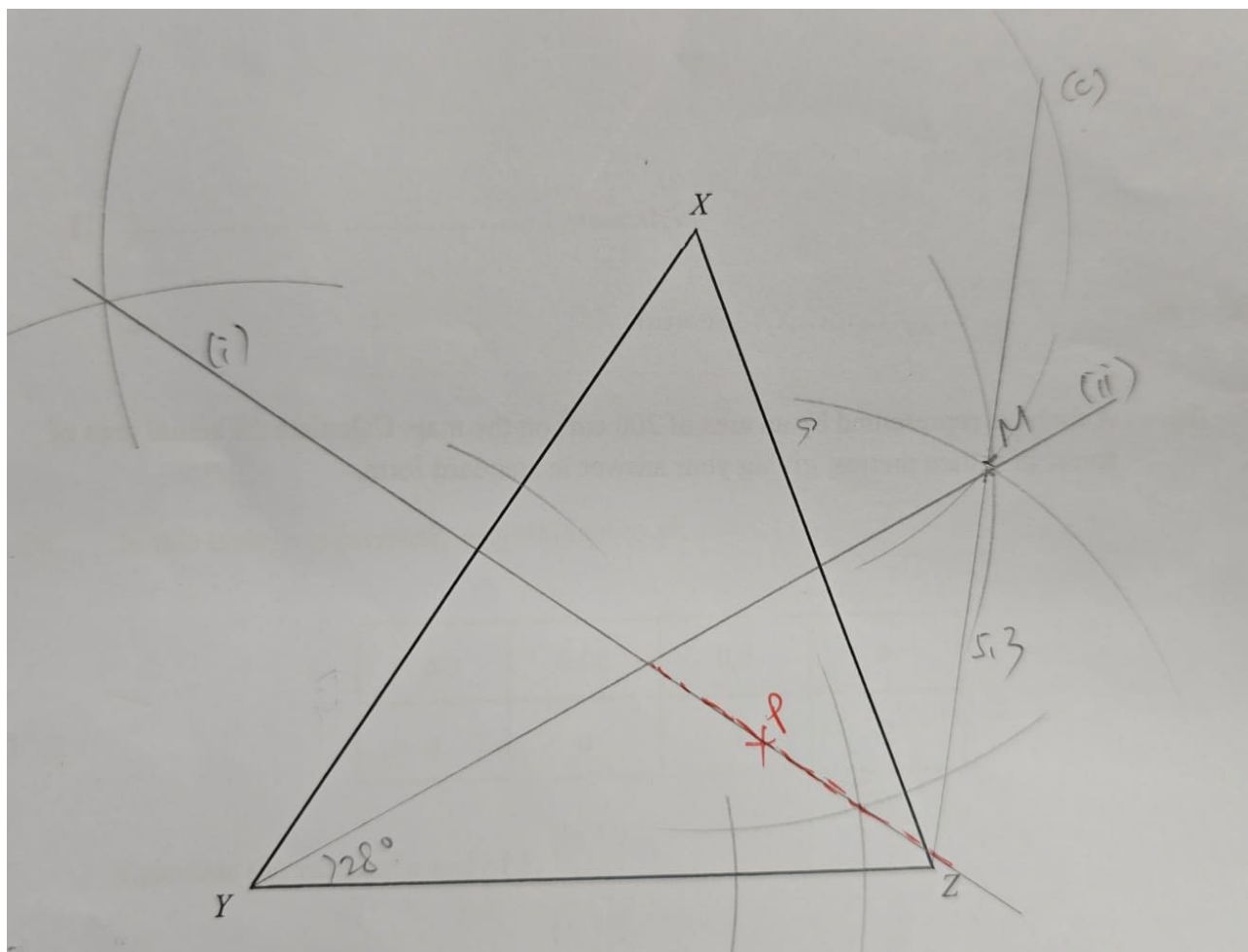
200 : 3125000

3.125×10^6

Answerm² [2]

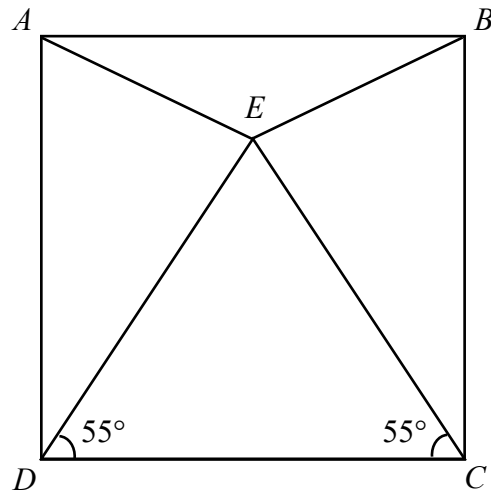
22 A triangle XYZ is shown below.

Answer



- (a) Construct
- (i) the perpendicular bisector of XY , [1]
 - (ii) the bisector of angle XYZ . [1]
- (b) Mark clearly a possible point which is inside the triangle, equidistant from X and Y , and nearer to YZ than XY . Label this point P . [1]
- (c) The point M is such that angle $YZM = 100^\circ$ and $XM = 5$ cm. Find the two possible positions of M and label them M_1 and M_2 . [2]

- 23 $ABCD$ is a square and angle $EDC = \text{angle } ECD = 55^\circ$.



Prove that triangle AEB is an isosceles triangle.

[4]

Mtd 1:

Since $\angle EDC = \angle ECD = 55^\circ$, $\triangle EDC$ is an isosceles triangle, $ED = EC$

$\angle ADE = \angle BCE = 90^\circ - 55^\circ = 35^\circ$

$AD = BC$ (sides of sq)

$\triangle ADE \equiv \triangle BCE$ (SAS)

$\therefore AE = BE$, $\triangle AEB$ is an isosceles triangle

Mtd 2:

Let P be the foot of the perpendicular from E to DC .

Since $\angle EDC = \angle ECD = 55^\circ$,

$\Rightarrow \triangle EDC$ is an isosceles triangle,

$\Rightarrow EP \perp DC$,

$\Rightarrow DP = PC$

Let EP extended to cut AB at Q .

Since $ABCD$ is a square and EP extended to Q on AB ,

$$\left. \begin{array}{l} \angle AQE = \angle BQE = 90^\circ \\ AQ = BQ \end{array} \right\}$$

EQ is a shared side for $\triangle QAE$ and $\triangle QBE$,

$\triangle QAE \equiv \triangle QBE$ (SAS)

$\therefore \angle QAE = \angle QBE$

$\Rightarrow \triangle AEB$ is an isosceles triangle.

Mtd 3:

$$\left. \begin{array}{l} \text{Since } \angle EDC = \angle ECD = 55^\circ, \\ \Rightarrow \triangle EDC \text{ is an isosceles triangle} \\ \Rightarrow E \text{ lies on the perpendicular bisector of } DC. \\ \text{Since } ABCD \text{ is a square,} \\ \Rightarrow E \text{ lies on the perpendicular bisector of } AB. \end{array} \right\}$$

Let Q = foot of the perpendicular bisector of AB from E .

$$\left. \begin{array}{l} \Rightarrow AQ = BQ \text{ and } \angle AQE = \angle BQE \\ EQ \text{ is a shared side for } \triangle QAE \text{ and } \triangle QBE, \end{array} \right\}$$

$\triangle QAE \equiv \triangle QBE$ (SAS)

$\therefore \angle QAE = \angle QBE$

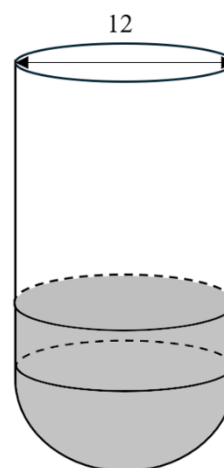
$\Rightarrow \triangle AEB$ is an isosceles triangle.

- 24 (a) A container is made from a hemisphere and cylinder.

The container has a diameter of 12 cm.

The container is filled with water such that the total surface area of water in contact with the container is 120π .

Find the volume of water in the container, leaving your answer in terms of π .



$$2\pi(6^2) + 2\pi(6)(h) = 120\pi$$

$$72\pi + 12\pi h = 120\pi$$

$$h = 4$$

$$\text{Vol} = \frac{2}{3}\pi(6)^3 + \pi(6^2)(4)$$

$$= 144\pi + 144\pi$$

$$= 288\pi$$

Answercm³ [4]

- (b) All the water in the container was poured into a cone with diameter 24 cm and a slant height of 20 cm. Calculate the depth, d , of the empty space.

Mtd 1:

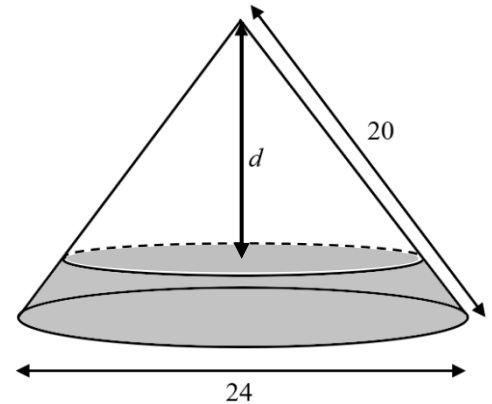
$$h = \sqrt{20^2 - 12^2} = 16$$

$$\begin{aligned} V_{air} &= \frac{1}{3} \pi (12^2)(16) - 288\pi \\ &= 768\pi - 288\pi \\ &= 480\pi \end{aligned}$$

$$\frac{V_{air}}{V_{cone}} = \left(\frac{d_{air}}{d_{cone}} \right)^3$$

$$\frac{480\pi}{768\pi} = \left(\frac{d_{air}}{16} \right)^3$$

$$\begin{aligned} d_{air} &= \sqrt[3]{\frac{480\pi}{768\pi}} (16) \\ &= 13.7 \end{aligned}$$



Mtd 2:

$$h = \sqrt{20^2 - 12^2} = 16$$

$$\frac{r}{12} = \frac{d}{16}$$

$$r = \frac{3}{4}d$$

$$\frac{1}{3} \pi (12^2)(16) - \frac{1}{3} \pi (r^2)(d) = 288\pi$$

$$768\pi - \frac{1}{3} \pi \left(\frac{3}{4}d \right)^2 (d) = 288\pi$$

$$\frac{9}{16} \pi d^3 = 480\pi$$

$$d^3 = 2560$$

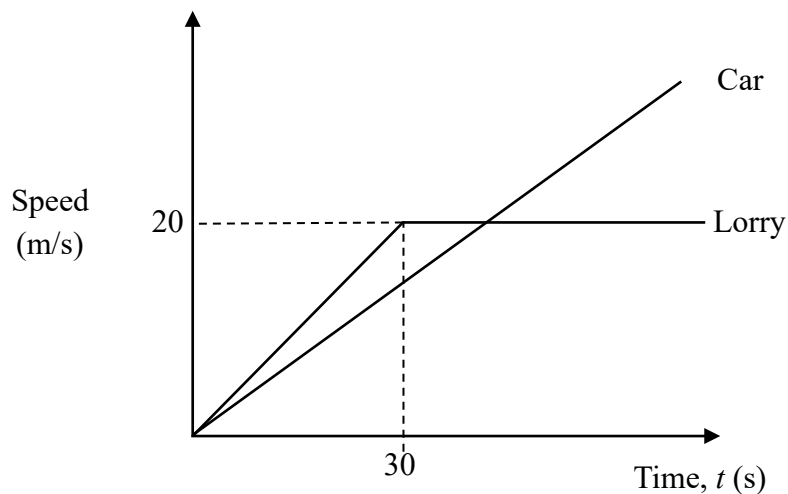
$$d_{air} = 13.7$$

Answer $d =$ _____ cm [4]

- 25** The diagram shows the speed-time graphs of a car and lorry traveling along the same road in the same direction. Both vehicles begin their journey from the same starting point.

The lorry accelerates for 30 seconds and then travels at a constant speed of 20 m/s.

The car accelerates at a uniform rate of 0.5 m/s^2 from rest.

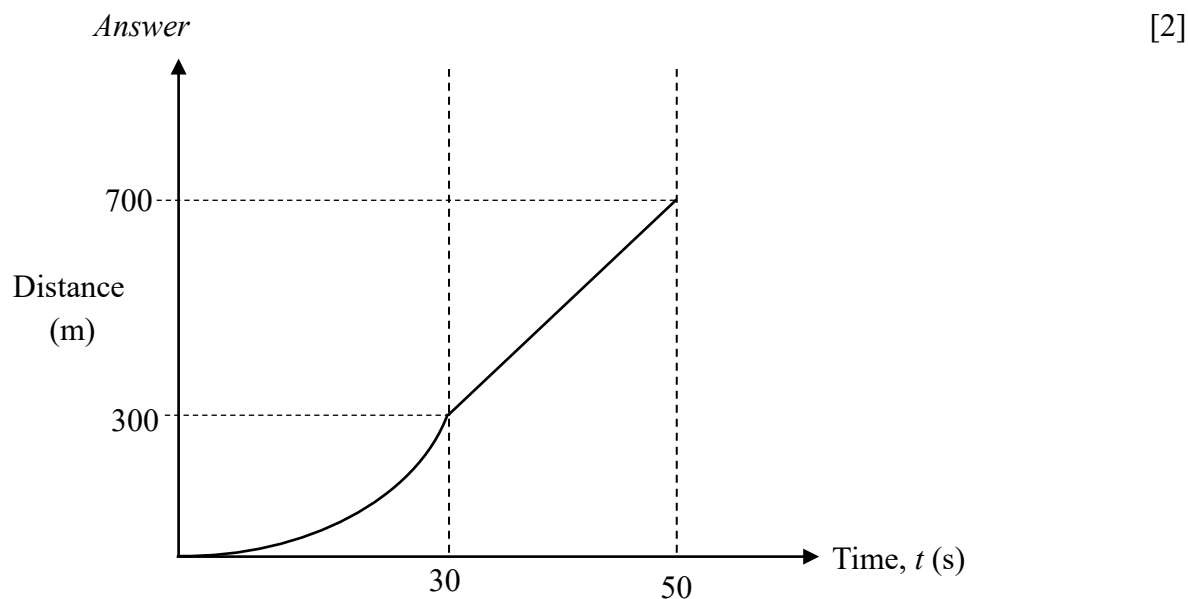


- (a)** Find the speed of lorry at 12 seconds.

$$\begin{aligned} \text{Speed} &= \frac{20}{30}(12) \\ &= 8 \text{ m/s} \end{aligned}$$

Answer _____ m/s [2]

- (b)** Draw the distance-time graph of the lorry for $0 \leq t \leq 50$ s.



- (c) Find the time taken for the car to meet the lorry.

Mtd 1:

$$V \text{ of car} = \frac{1}{2}t$$

$$\text{Dist traveled by car} = \frac{1}{2}\left(\frac{1}{2}t\right)(t)$$

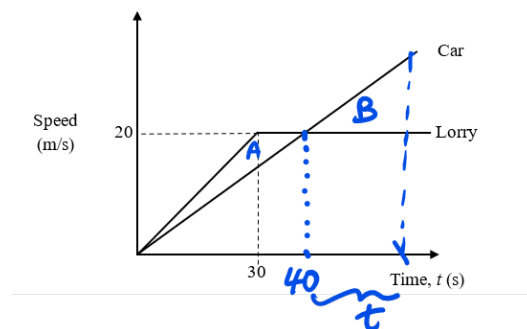
$$\text{Dist traveled by lorry} = 300 + 20(t - 30)$$

$$\frac{1}{2}\left(\frac{1}{2}t\right)(t) = 20(t) - 300$$

$$\frac{1}{4}t^2 - 20t + 300 = 0$$

$$t = 60 \quad \text{or} \quad t = 20(\text{rej})$$

Mtd 2:



$$V \text{ of car} = \frac{1}{2}t$$

$$\text{When } v = 20, \frac{1}{2}t = 20 \Rightarrow t = 40$$

$$\text{Area of region A} = \frac{1}{2}(40 - 30)(20) = 100$$

$$\text{Area of region B} = \frac{1}{2}\left(\frac{1}{2}t\right)t$$

$$\frac{1}{2}\left(\frac{1}{2}t\right)(t) = 100$$

$$t^2 = 400$$

$$t = 20$$

$$\text{Time taken } 40 + 20 = 60$$

Answer _____ s [4]

Name:		Index Number:		Class:	
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SOLUTIONS

2025 Preliminary Examination Secondary 4 (O-Level Programme)



Mathematics

4052/02

Paper 2

27 Aug 2025

2 hours 15 minutes

Booklet A

Additional material: Question Booklet B

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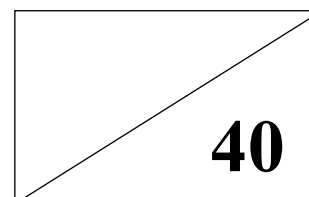
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For examiners' use



Mathematical Formulae

Compound interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4 \pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} a b \sin C$$

$$\text{Arc length} = r \theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2 b c \cos A$$

Statistics

$$\text{Mean} = \frac{\sum f x}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum f x^2}{\sum f} - \left(\frac{\sum f x}{\sum f} \right)^2}$$

- 1 Every evening Juline and Evangeline play a game of tic-tac-toe. When they play against each other, Juline has a $\frac{2}{5}$ chance of winning and Evangeline has a $\frac{1}{6}$ chance of winning. Find, as a fraction in its simplest form, the probability that
- (i) Juline will win on both Monday and Tuesday next week.

$$P(\text{Juline wins both days})$$

$$= \frac{2}{5} \times \frac{2}{5}$$

$$= \frac{4}{25}$$

Answer _____ [1]

- (ii) Juline will win on either Wednesday or Thursday next week but not on both days.

$$P(\text{Juline wins one day only}) = P(J) \times P(\text{Not } J) + P(\text{Not } J) \times P(J)$$

$$\left(\frac{2}{5} \times \frac{3}{5} \right) \times 2$$

$$= \frac{12}{25}$$

Answer _____ [2]

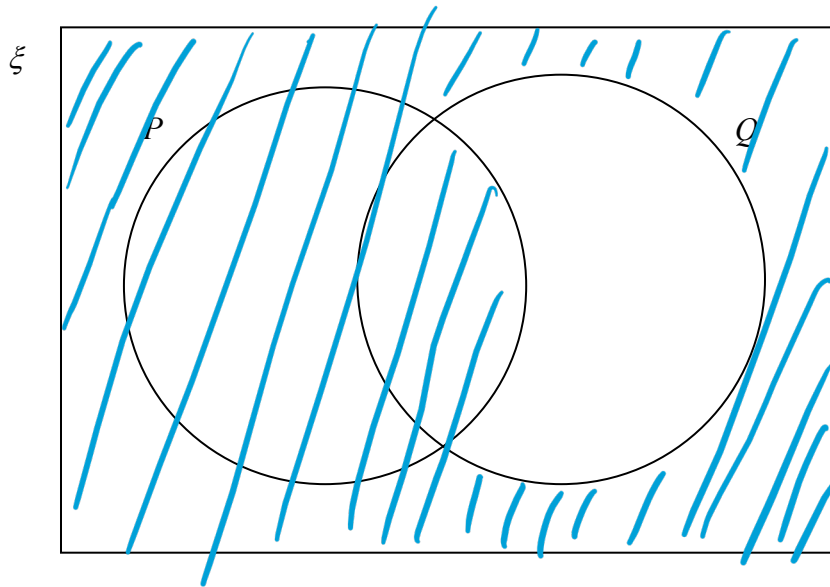
- (iii) it will be a draw on Friday next week.

$$P(\text{Draw}) = 1 - \frac{2}{5} - \frac{1}{6}$$

$$= \frac{13}{30}$$

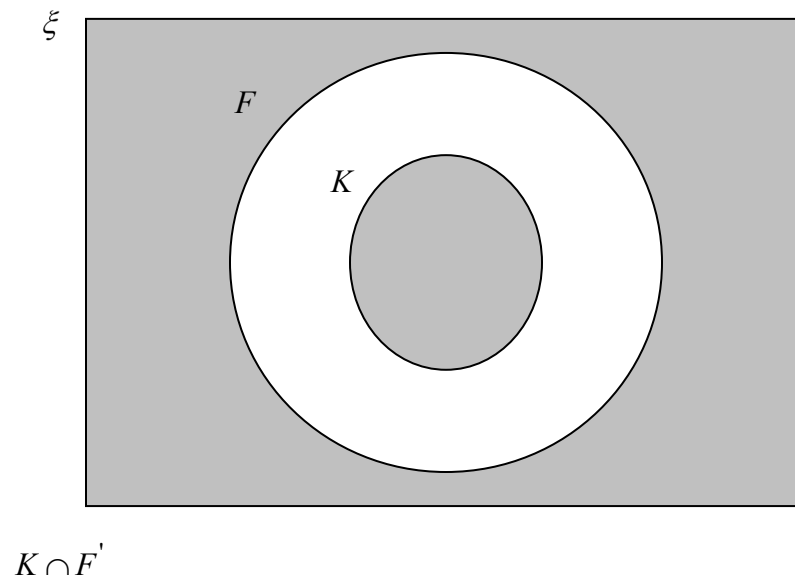
Answer _____ [2]

2(a)(i) On the Venn diagram, shade the region which represents $(P' \cap Q)'$.



[1]

(ii) Use set notation to describe the shaded region.



Answer _____ [1]

2(b) $\xi = \{\text{integers } x : 20 \leq x \leq 50\}$

$A = \{\text{perfect square}\}$

$B = \{\text{prime numbers}\}$

$C = \{\text{multiples of 7}\}$

(i) List the elements in $A \cap C'$.

25, 36

Answer _____[1]

(ii) Find the value of $n(B \cap C)$.

0

Answer _____[1]

(iii) A number, p , is chosen at random from the set $(B \cup C)$.
Find the probability that $p \in A$.

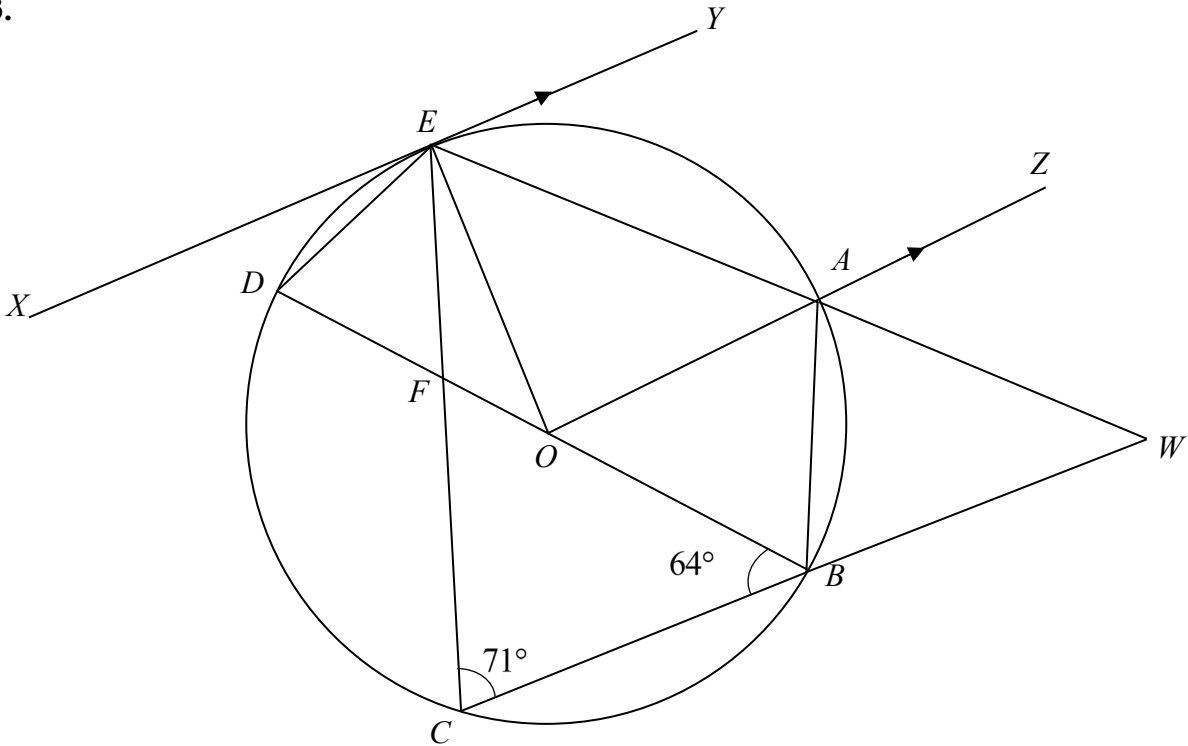
$$B \cup C = \{21, 23, 28, 29, 31, 35, 37, 41, 42, 43, 47, 49\}$$

$$n(B \cup C) = 12 \quad [\text{recognise that 49 is an element that belongs to both } B \cup C \text{ and } A]$$

$$\text{Required probability is } \frac{1}{12}$$

Answer _____[2]

3.



O is the centre of the circle $ABCDE$. XY is a tangent to the circle at E and EC intersects BD at F . OZ is a straight line that passes through A and is parallel to XY . The lines CB produced and EA produced meet at W . $\angle BCE = 71^\circ$ and $\angle OBC = 64^\circ$.

(a) Complete the following statements.

(i) $\angle BDE =$ _____ 71 _____ $^\circ$

because _____ angles in the same segment _____ [2]

(ii) $\angle AOE =$ _____ 90 _____ $^\circ$

because ...tangent is perpendicular to radius , interior angles between parallel lines $OZ \parallel XY$[2]

Find

(b) $\angle BOE$,

$$\begin{aligned}\angle BOE &= 71^\circ \times 2 \\ &= 142^\circ\end{aligned}$$

Answer $\angle BOE =$ _____ $^\circ$ [1]

(c) $\angle OCE$,

$$\begin{aligned}\angle EOC &= 71^\circ - 64^\circ \\ &= 7^\circ\end{aligned}$$

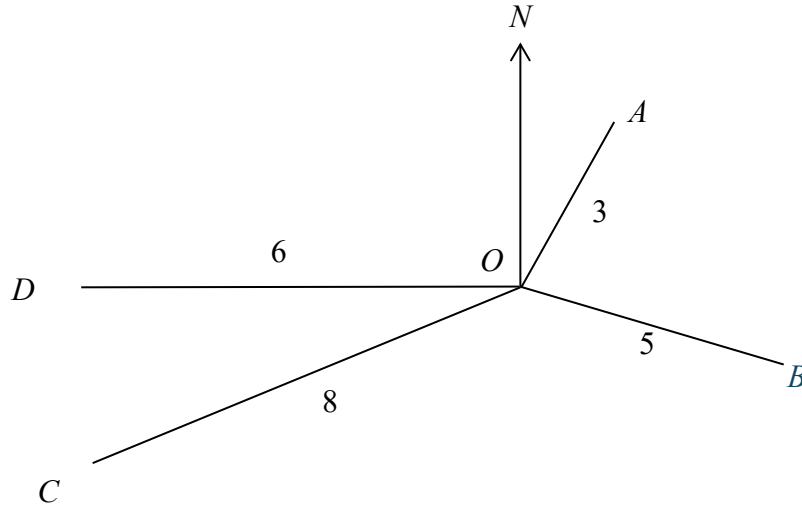
Answer $\angle OCE =$ _____ $^\circ$ [1]

(d) $\angle ABW$.

$$\begin{aligned}\angle OEA &= \frac{180^\circ - 90^\circ}{2} = 45^\circ \\ \angle ABW &= \angle AEC \text{ (exterior angle of cyclic quadrilateral)} \\ &= 45^\circ + 7^\circ \\ &= 52^\circ\end{aligned}$$

Answer $\angle ABW =$ _____ $^\circ$ [2]

4. In the diagram, a lighthouse is at O .
 Four ships, A , B , C and D are positioned around it with $OA = 3$ km, $OB = 5$ km, $OC = 8$ km and $OD = 6$ km. The bearing of A from O is 027° and the bearing of B from O is 103° . Ship D lies due west of O and Ship C lies due south of D .



- (a) Calculate the distance between Ship A and Ship B .

Using the Cosine Rule:

$$\angle AOB = 103^\circ - 27^\circ = 76^\circ$$

$$AB^2 = 3^2 + 5^2 - 2(3)(5)\cos(76^\circ)$$

$$= 9 + 25 - 30 \times 0.241921$$

$$= 26.74235$$

$$AB = 5.1713$$

$$= 5.17 \text{ km}$$

Answer _____ km [3]

- (b) The keeper of the lighthouse is standing on top of the lighthouse, which is 20 m tall. Calculate the angle of depression, from where he is standing to Ship *A*.

$$\begin{aligned}\tan \theta &= \frac{20}{3000} \\ \theta &= 0.381966 \\ &= 0.381^\circ \\ &= 0.4^\circ\end{aligned}$$

Answer _____ ° [2]

- (c) Calculate the bearing of Ship *C* from *O*.

$$\begin{aligned}\cos \angle COD &= \frac{6}{8} \\ \angle COD &= 41.4096^\circ\end{aligned}$$

$$\begin{aligned}\text{Bearing of } C \text{ from } O &\text{ is } 270^\circ - 41.4096^\circ \\ &= 228.5904^\circ \\ &= 228.6^\circ\end{aligned}$$

Answer ° [2]

- (d) A speedboat travels directly from Ship A to Ship B . Calculate the shortest distance between the speedboat and the lighthouse during its journey.

Let the shortest distance be x km

$$\frac{1}{2}x(5.1713) = \frac{1}{2}(5)(3)\sin 76^\circ$$

$$x = 2.81446$$

$$= 2.81$$

Answer _____ km [2]

- (e) It is suggested that Ships A , B and the lighthouse at O form a right-angled triangle. Showing your working clearly, determine whether this is true.

$$\text{Given } OA = 3, OB = 5$$

$$\text{from (a), } AB = 5.1713$$

$$OA^2 + OB^2 = 3^2 + 5^2 = 34 \neq AB^2$$

Conclusion:

Since Pythagoras' Thm does not applied, triangle OAB is not a right-angled triangle hence Ships A , B and the lighthouse at O do not form a right-angled triangle.

“Pythagoras' theorem” must be seen, before concluding that OAB is not a right-angle triangle.

Answer

[2]

- 5. (a)** As part of a school fundraising event, Jamie buys x boxes of snacks for a total of \$180. She plans to sell each box at a profit of 80 cents.

(i) Write, in terms of x , an expression for the price, in dollars, Jamie paid for one box.

$$\text{Cost per box} = \frac{180}{x}$$

Answer \$_____ [1]

(ii) Hence, write an expression for her selling price, in dollars, of one box.

$$\text{Selling price per box} = \frac{180}{x} + 0.8$$

Answer \$_____ [1]

- (b)** Jamie managed to sell all except 15 of the boxes and collected \$171 from his sales. Form an equation in x , and show that it simplifies to $4x^2 - 15x - 13500 = 0$.

$$\left(\frac{180}{x} + 0.8 \right) (x - 15) = 171$$

$$180 - \frac{2700}{x} + 0.8x - 12 = 171$$

$$0.8x^2 - 2700 - 3x = 0$$

$$8x^2 - 30x - 27000 = 0$$

$$4x^2 - 15x - 13500 = 0 \text{ (Shown)}$$

Answer Shown above [3]

- (c) Showing your working clearly, solve the equation in (b) and calculate the selling price of 1 box of snacks.

$$4x^2 - 15x - 13500 = 0$$

Using general formula,

$$x = \frac{-(-15) \pm \sqrt{(-15)^2 - 4(4)(-13500)}}{2(4)}$$

$$= \frac{15 \pm \sqrt{225 + 216000}}{16}$$

$$x = \frac{15 \pm 465}{8}$$

$$x = 60 \quad \text{or} \quad x = -56.25 \text{ (reject)}$$

$$\text{Selling price} = \frac{180}{60} + 0.8$$

$$= 3.80$$

Answer $x =$ _____

Selling price = \$_____ [3]

- (d) Jamie considers increasing her selling price by \$0.25 per box for the remaining 15 boxes. Would this allow her to make a profit overall?
Justify your conclusion with mathematical calculations.

$$\text{New selling price} = 3.80 + 0.25 = 4.05$$

$$\text{Total revenue} = 171 + 15 \times 4.05 = \$231.75 > \$180$$

Yes, Jamie's claim is correct. By increasing her selling price per box, she would raise a profit as the revenue collected is more than \$180, the amount she spent.

Answer

[2]

END OF BOOKLET A

Name:		Index Number:		Class:	
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**2025 Preliminary Examination
Secondary 4 (O-Level Programme)**

B

Mathematics

Paper 2

4052/02

27 August 2025

Booklet B

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

For examiners' use

50

This booklet consists of **17** printed pages and **1** blank pages.

- 6.(a)** Complete the table of values for $y = \frac{1}{10}\left(x^2 + \frac{7}{x}\right)$.

Values are given to one decimal place where appropriate.

x	0.6	1	1.5	2.2	3.1	4	5.1	5.8
y	1.2	0.8	0.7	0.8		1.8	2.7	3.5

[1]

$$p = 1.2 \quad B1$$

- (b)** On the grid opposite, draw the graph of $y = \frac{1}{10}\left(x^2 + \frac{7}{x}\right)$ for $0.6 \leq x \leq 5.8$. [3]

- (c)** Using your graph, estimate the value(s) of x for which $x^2 + \frac{7}{x} = 20$, where $0.6 \leq x \leq 5.8$.

$$x^2 + \frac{7}{x} = 20$$

$$\div 10$$

$$\frac{1}{10}\left(x^2 + \frac{7}{x}\right) = 2$$

$$\text{Draw } y = 2$$

$$x = 4.25$$

$$[\text{Accept } 4.15 \text{ to } 4.35]$$

$$\text{Answer } x = \text{_____} [2]$$

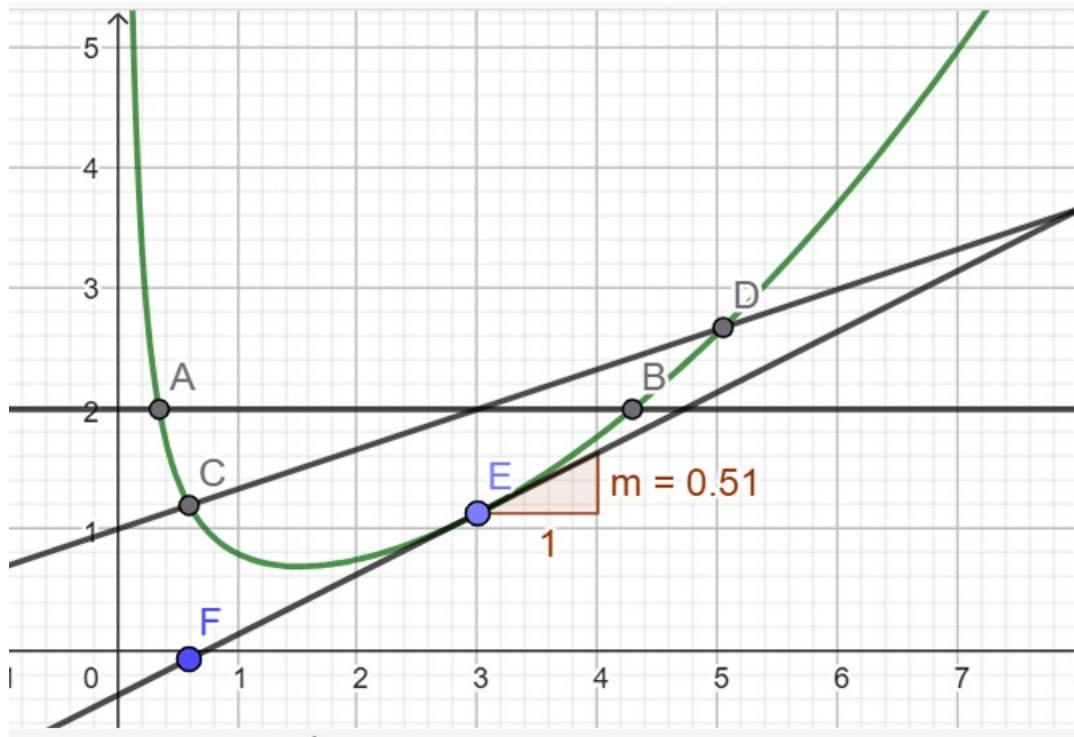
- (d) (i)** On the same grid, draw the graph of $3y - x = 3$ for $0.6 \leq x \leq 5.8$. [1]

$$\text{Draw } y = \frac{1}{3}x + 1$$

- (ii)** Write down the x -coordinates of the points where the line intersects the curve.

$$x = 0.6 \quad \text{or} \quad x = 5.05$$

$$\text{Answer } x = \text{_____} \text{ and } x = \text{_____} [2]$$



- (iii) These values of x are solutions of the equation $3x^3 + Ax^2 + Bx + 21 = 0$. Find the value of A and of B .

$$\begin{aligned}\frac{1}{10}\left(x^2 + \frac{7}{x}\right) &= \frac{1}{3}x + 1 \\ x^2 + \frac{7}{x} &= \frac{10x}{3} + 10 \\ x^3 - \frac{10x}{3} - 10 + \frac{7}{x} &= 0 \\ 3x^3 - 10x - 30 + \frac{21}{x} &= 0 \\ 3x^3 - 10x^2 - 30x + 21 &= 0 \\ A &= -10 \\ B &= -30\end{aligned}$$

Answer $A =$ _____

$B =$ _____ [3]

- (e) By drawing a tangent, find the gradient of the curve where $x = 2.2$.

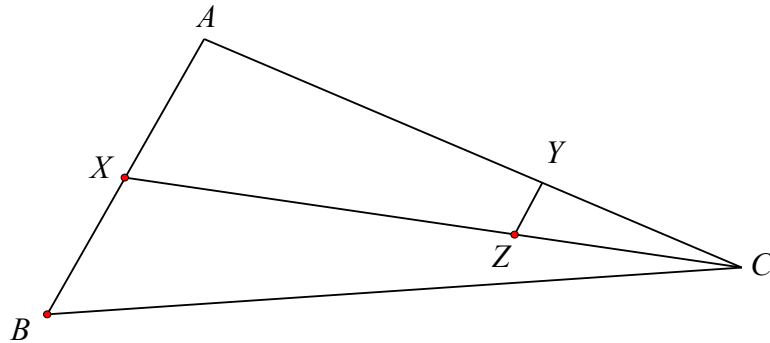
$$\begin{aligned}\text{Gradient} &= \frac{1.25 - 0.35}{3.5 - 1} \\ &= 0.36 \\ &[\text{Accept } 0.26 < m < 0.375]\end{aligned}$$

Answer _____[2]

7. ABC is a triangle. $\overrightarrow{AB} = \mathbf{p}$ and $\overrightarrow{AC} = \mathbf{q}$. X is the midpoint of AB . Z and Y are points on

XC and AC respectively such that $\overrightarrow{XZ} = \frac{2}{3}\overrightarrow{XC}$ and $\overrightarrow{YZ} = k\overrightarrow{AC}$.

[minus 1 mark for whole question if no vector notations]



- (a) Express the following vectors in terms of $\mathbf{p}$ and/or $\mathbf{q}$, giving your answers in the simplest terms.

- (i) $\overrightarrow{AX}$,

$$\frac{1}{2}\mathbf{p}$$

Answer $\overrightarrow{AX} =$ _____ [1]

- (ii) $\overrightarrow{CX}$.

$$\begin{aligned}\overrightarrow{CX} &= \overrightarrow{AX} - \overrightarrow{AC} \\ &= \frac{1}{2}\mathbf{p} - \mathbf{q}\end{aligned}$$

Answer $\overrightarrow{CX} =$ _____ [2]

- (b) Express $\overrightarrow{YZ}$ in terms of k , $\mathbf{p}$ and $\mathbf{q}$.

$$\begin{aligned}\overrightarrow{YZ} &= \overrightarrow{YC} + \overrightarrow{CZ} \\ &= k\mathbf{q} + \frac{1}{3}\overrightarrow{CX} \\ &= k\mathbf{q} + \frac{1}{3}\left(\frac{1}{2}\mathbf{p} - \mathbf{q}\right) \\ &= \frac{1}{6}\mathbf{p} + \left(k - \frac{1}{3}\right)\mathbf{q}\end{aligned}$$

Answer $\overrightarrow{YZ} =$ _____ [2]

(c) Given that YZ is parallel to AX , find the numerical value of

(i) k ,

$$\frac{1}{3}$$

Answer $k =$ _____ [1]

(ii) $\frac{YZ}{AB}$,

$$\frac{1}{6}$$

Answer $\frac{YZ}{AB} =$ [1]

(iii) $\frac{\text{Area of } \triangle XYZ}{\text{Area of } \triangle ABC}$.

$$\begin{aligned} \frac{\text{Area of } \triangle XYZ}{\text{Area of } \triangle AXC} &= \frac{1}{2} \\ \frac{\text{Area of } \triangle XYZ}{\text{Area of } \triangle ABC} &= \frac{1}{3+3} \\ &= \frac{1}{6} \end{aligned}$$

Answer $\frac{\text{Area of } \triangle XYZ}{\text{Area of } \triangle ABC} =$ _____ [1]

8. A manufacturer sells lightbulbs under two different brands, **BrightLite** and **EcoGlow**. The lifespan, in days, of **500 BrightLite lightbulbs** is summarised in the table below.

Lifespan of **BrightLite Lightbulbs**

Length of life (x days)	Number of bulbs
$100 < x \leq 120$	15
$120 < x \leq 140$	35
$140 < x \leq 160$	70
$160 < x \leq 180$	130
$180 < x \leq 200$	150
$200 < x \leq 220$	80
$220 < x \leq 240$	20

- (a) (i) Calculate an estimate of the mean lifespan of the **BrighLite bulbs** in days.

$$\begin{aligned}
 \text{Mean} &= \frac{15 \times 110 + 35 \times 130 + 70 \times 150 + 130 \times 170 + 150 \times 190 + 80 \times 210 + 20 \times 230}{500} \\
 &= 177.4 \text{ days}
 \end{aligned}$$

Answer _____ days [1]

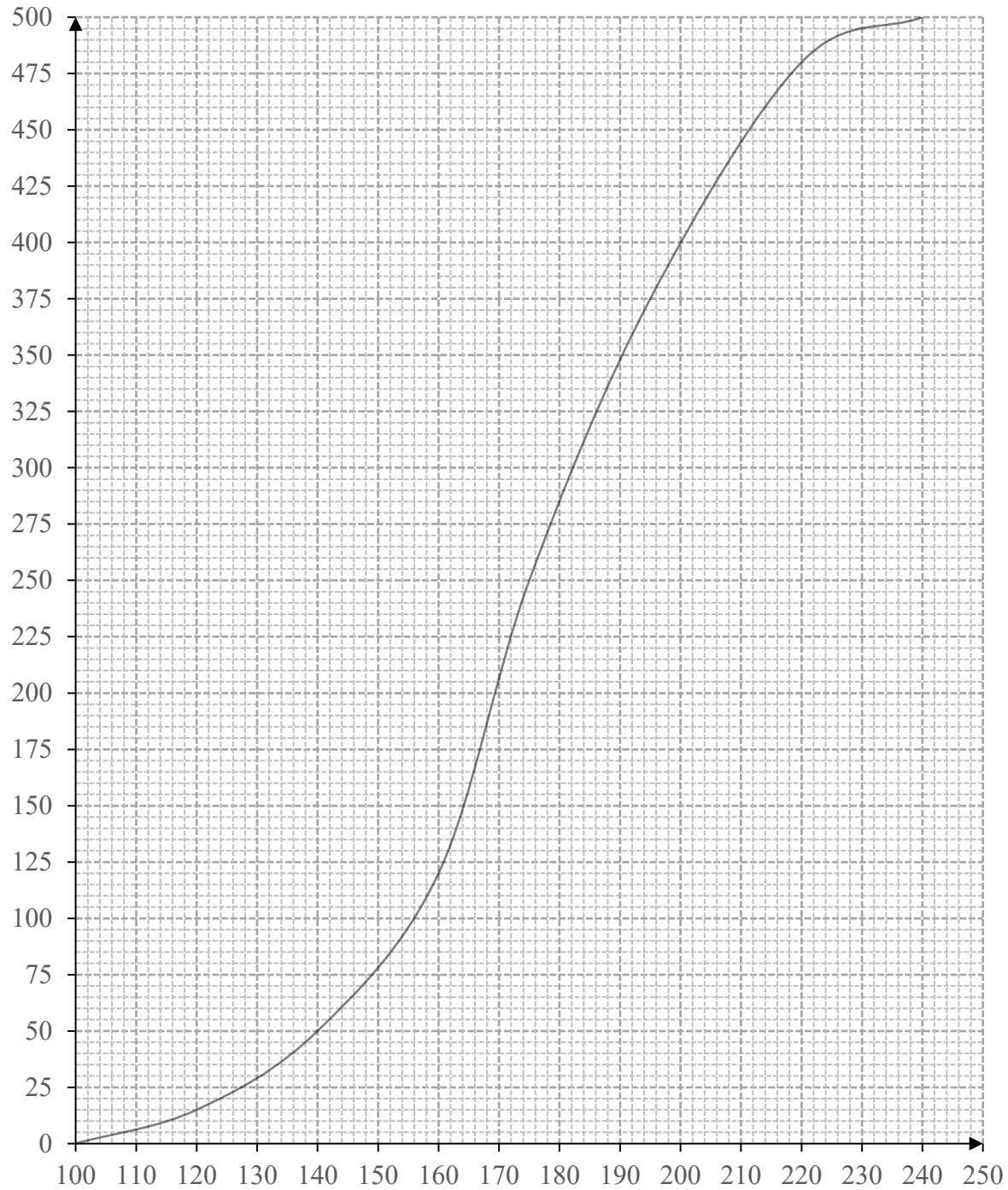
- (ii) Calculate an estimate of the standard deviation.

$$\text{Standard deviation} = 27.2 \text{ days}$$

Answer _____ days [1]

(b) The cumulative frequency curve shows the same information for the 500 **BrightLite** **Bulbs**.

Cumulative Frequency



Lifespan of Lightbulbs in Days

Use the curve to estimate

- (i) the median lifespan of the bulbs,

$$\text{Median} = 175 \text{ days}$$

Answer _____ days [1]

- (ii) the interquartile range of the lifespans.

$$\begin{aligned}\text{Interquartile range} &= (195 - 161) \text{ days} \\ &= 34 \text{ days}\end{aligned}$$

Answer _____ days [2]

- (iv) If a bulb is selected, find the probability that its lifespan is more than 160 days.

$$\begin{aligned}P(\text{lifespan} > 160 \text{ days}) &= \frac{380}{500} \\ &= \frac{19}{25}\end{aligned}$$

Answer _____ [2]

- (iv) 90% of the bulbs have lifespan more than n days. Find n .

$$n = 140 \text{ days}$$

Answer $n =$ _____ [1]

- (v) If two bulbs are randomly chosen, find the probability that one bulb has a lifespan of more than 160 days while the other has a lifespan of less than 140 days.

$P(\text{one has lifespan} > 160 \text{ days and one has lifespan} < 140 \text{ days}) =$

$$\begin{aligned} & \frac{380}{500} \times \frac{50}{499} + \frac{50}{500} \times \frac{380}{499} \\ &= \frac{76}{499} \end{aligned}$$

Answer _____[2]

- (c) The manufacturer also tested 500 **EcoGlow bulbs**. These bulbs had a smaller interquartile range as the **BrightLite bulbs**, but the same median lifespan. Describe how the cumulative frequency curve for the **EcoGlow bulbs** may differ from that of the **BrightLite bulbs**.

The **EcoGlow curve** will be **steeper** than the curve of the BrightLite bulbs.

Answer

[1]

9. A company produces four types of healthy snacks — granola bars, fruit juices, protein shakes, and yoghurt cups. The quantities delivered daily to three health stores are shown in the table below.

Snack Type	Granola Bars	Fruit Juices	Protein Shakes	Yoghurt Cups
Store A	45	30	25	20
Store B	50	25	30	35
Store C	60	40	20	25

[Order of matrix must be correct]

- (a) Write down a 3×4 matrix P to show the number of each type of snack delivered to each store.

$$P = \begin{pmatrix} 45 & 30 & 25 & 20 \\ 50 & 25 & 30 & 35 \\ 60 & 40 & 20 & 25 \end{pmatrix}$$

Answer $P =$ _____[1]

- (b) The cost of each granola bar, fruit juice, protein shake, and yoghurt cup is \$1.50, \$2.00, \$3.20 and \$1.80 respectively. Represent these prices in a 4×1 column matrix Q .

$$Q = \begin{pmatrix} 1.50 \\ 2 \\ 3.20 \\ 1.80 \end{pmatrix}$$

Answer $Q =$ _____[1]

(c) Evaluate the matrix $T = PQ$.

$$\begin{aligned}
 PQ &= \begin{pmatrix} 45 & 30 & 25 & 20 \\ 50 & 25 & 30 & 35 \\ 60 & 40 & 20 & 25 \end{pmatrix} \begin{pmatrix} 1.50 \\ 2 \\ 3.20 \\ 1.80 \end{pmatrix} \\
 &= \begin{pmatrix} 45 \times 1.5 + 30 \times 2 + 25 \times 3.2 + 20 \times 1.80 \\ 50 \times 1.5 + 25 \times 2 + 30 \times 3.2 + 35 \times 1.80 \\ 60 \times 1.5 + 40 \times 2 + 20 \times 3.2 + 25 \times 1.80 \end{pmatrix} \\
 &= \begin{pmatrix} 243.50 \\ 284 \\ 279 \end{pmatrix}
 \end{aligned}$$

Answer $T =$ _____[2]

(d) State what each of the elements in matrix T represents.

Each element in matrix PQ represents the **total value of items delivered** (in dollars) to each store **respectively**.

Answer

[1]

(e) (i) Find the matrix $\begin{pmatrix} 1 & 1 & 1 \end{pmatrix} P$.

$$\begin{aligned}
 \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} P &= \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 45 & 30 & 25 & 20 \\ 50 & 25 & 30 & 35 \\ 60 & 40 & 20 & 25 \end{pmatrix} \\
 &= \begin{pmatrix} 45 + 50 + 60 & 30 + 25 + 40 & 25 + 30 + 20 & 20 + 35 + 25 \end{pmatrix} \\
 &= \begin{pmatrix} 155 & 95 & 75 & 80 \end{pmatrix}
 \end{aligned}$$

Answer _____[1]

(ii) Explain what each element in $(1 \ 1 \ 1)P$ represents.

Each element represents the total number of Granola Bars, Fruit Juices, Protein Shakes and Yogurt cups **respectively** delivered to all three stores.

Note: respectively must be used for the Granola Bars, Fruit Juices, Protein Shakes and Yogurt cups

Answer

[1]

10. Alex is starting a vertical farming business in Singapore, using hydroponics to grow leafy vegetables in an indoor warehouse space of floor area 50 m² with a ceiling height of 2.5 metres. The current setup consists of 8 vertical hydroponic shelves arranged within the space.

Fixed Overheads and Costs:

- Electricity: \$0.25 per kWh
- Water: \$2.50 per m³
- Nutrients: \$0.30 per kg of vegetables grown

The business rents a government-subsidised commercial unit at a special rate of **\$500/month**, under a national initiative to support sustainable urban agriculture and small business growth.

The table below gives information on the Monthly Production, Unit price, and Resource Usage for January to June 2025.

			Resource Usage	
Month	Monthly production (kg)	Price (\$/kg)	Electricity (kWh)	Water (m ³)
Jan	240	6.00	800	15
Feb	260	6.10	820	16
Mar	280	6.20	840	17
Apr	300	6.30	860	18
May	320	6.40	880	19
Jun	340	6.50	900	20

(a) Calculate the profit for the month of June.

$$\text{Revenue} = \text{Production} \times \text{Price} = 340 \text{ kg} \times \$6.50 = \$2,210.$$

Costs:

- Electricity: $900 \text{ kWh} \times \$0.25 = \225

- Water: $20 \text{ m}^3 \times \$2.50 = \50

- Nutrients: $340 \text{ kg} \times \$0.30 = \102

- Rent: \$500

$$\text{Total Cost} = \$877$$

$$\text{Profit} = \text{Revenue} - \text{Total Cost} = \$2210 - \$877 = \$1333.$$

Answer: \$1333

$$\begin{aligned} \text{June Profit} &= 340 \times 6.50 - (500 + 225 + 50 + 102) \\ &= \$1333 \end{aligned}$$

Answer \$ _____[3]

(b) Suggest a possible reason for the increase in crop prices from January to June.

Plausible reasons:

demand grew faster than supply (e.g., new restaurant contracts or a growing subscription base), so even as output rose each month, crops still selling out—letting Alex **nudge the price up gradually** from \$6.00 to \$6.50 without losing sales.

positioning as premium/local pesticide-free produce

passing on higher operating costs like electricity or labour

inflation

to increase profits

Answer

[1]

- (c) The vertical farming system uses hydroponic shelves, each measuring 2.0 m (length) $\times$ 0.5 m (width) $\times$ 1.8 m (height), with 5 levels of planting trays per shelf. Each level grows 10 kg of vegetables per month. There are currently 8 such shelves arranged in the warehouse. Safety guidelines require a minimum clearance of 0.2 m from the ground and 0.5 m from the ceiling.

The maximum number of shelves that can be installed is limited by both walking and maintenance space requirements. Each shelf occupies **2.0 m $\times$ 0.5 m** of floor space, and minimum clear walking space of **0.5 m** must be provided around each shelf. This clearance applies to the warehouse walls and between shelves.

The warehouse has a total floor area of **50 m²** (10 m $\times$ 5 m).

Due to airflow and light distribution limits, the maximum number of shelves that can physically fit in the warehouse is limited.

- For **10 shelves or fewer**, each shelf operates at **100 %** of its production capacity.
- When **11 or more shelves** are installed, airflow saturation occurs, reducing the production of **all** shelves to **85 %** of their monthly production.
- In addition, **each shelf beyond the 10th** operates at only **70 %** of its monthly production due to poor light and air mixing at the edges.

Alex's goal is to **maximise revenue** for the month of July. Assume that the unit price and resource usage for July is the same as June.

Task:

Determine the number of shelves Alex should install. Justify your answer clearly with mathematical calculations that consider both **space constraints** and the **productivity limitations**. You may wish to draw diagrams to aid in your understanding.

[6]

Continue with solutions for 10(c)

Students to show at least 2 orientations so that they could compare

Orientation A – Long side of shelf along length

Along the 10 m length:

$$0.5 + 2.0 + 0.5 + 2.0 + 0.5 + 2.0 + 0.5 = 8.0 \text{ m (3 shelves) } \checkmark$$

$$\text{Adding a 4th shelf would be: } 0.5 + 2.0 + 0.5 + 2.0 + 0.5 + 2.0 + 0.5 + 2.0 + 0.5 = 10.5 \text{ m } \times$$

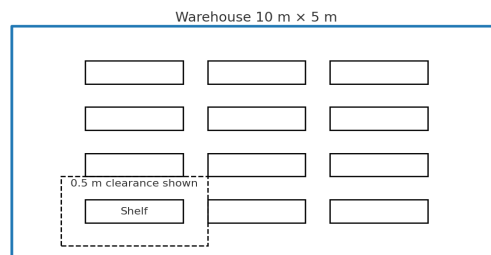
Along the 5 m width:

$$0.5 + (0.5 + 0.5 + 0.5 + 0.5) + 0.5 = 4.5 \text{ m (4 shelves) } \checkmark$$

$$\text{Adding a 5th shelf would be: } 0.5 + (5 \times 0.5) + 0.5 = 5.5 \text{ m } \times$$

Total by space = $3 \times 4 = 12$ shelves.

Orientation A (2.0 m $\times$ 0.5 m): Fits $3 \times 4 = 12$ shelves with 0.5 m gaps/margins



Orientation B – Long side of shelf along width of room

Along the 10 m length (each shelf uses 0.5 m length):

$$0.5 + 9 \times 0.5 + 8 \times 0.5 + 0.5 = 0.5 + 4.5 + 4.0 + 0.5 = 9.5 \text{ m (9 shelves) } \checkmark$$

$$\text{Trying 10 shelves: } 0.5 + 10 \times 0.5 + 9 \times 0.5 + 0.5 = 0.5 + 5.0 + 4.5 + 0.5 = 10.5 \text{ m } \times$$

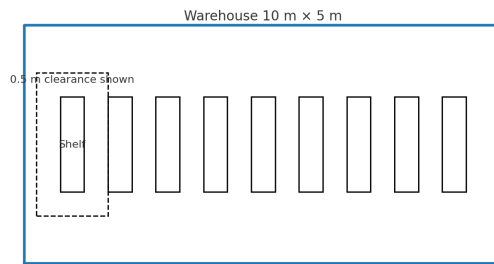
Along the 5 m width (each shelf uses 2.0 m width):

$$0.5 + 2.0 + 0.5 = 3.0 \text{ m (1 shelf across the width) } \checkmark$$

$$\text{Trying 2 across: } 0.5 + 2.0 + 0.5 + 2.0 + 0.5 = 5.5 \text{ m } \times$$

Total by space = $9 \times 1 = 9$ shelves.

Orientation B (0.5 m × 2.0 m): Fits $9 \times 1 = 9$ shelves with 0.5 m gaps/margins



Orientation C (combination of vertical and horizontal)

15 Shelves

Output and productivity per-shelf (Accept if students worked out only output to compare)

Per shelf baseline: 5 levels × 10 kg = 50 kg.

Congestion penalty (given)

- Up to 10 shelves: 50 kg each (100%).
- For 11 or more shelves: first 10 at 42.5 kg each (85%); each shelf beyond 10 at 35 kg (70%).

Revenue Comparison (Price = \$6.50/kg)

Orientation A (can reach 12 shelves):

Number of shelves	Output	Revenue	Best
8	400	2600	
9	450	2925	
10	500	3250	😊
11	460	2990	
12	495	3217	

Best with A: 10 shelves

Orientation B (space cap 9 shelves):

Number of shelves	Output	Revenue	Best
8	400	2600	
9	450	2925	😊

Best with B: 9 shelves

Orientation C (space cap of 15 shelves)

Number of shelves	Output	Revenue	Best
13	530		
14	565		
15	600	3900	😊

Conclusion (by output or revenue)

Orientation A allows up to 12 shelves by space but **10 shelves** gives the highest revenue: $500 \text{ kg} \times \$6.50 = \$3,250$

Orientation B fits at most **9 shelves**, giving at most \$2,925

Final Answer: Install 10 shelves (Orientation A).

Orientation C allows up to **15 shelves** by space and 15 shelves gives the highest revenue

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