

PRELIMINARY EXAMINATION 2025
SECONDARY FOUR

CANDIDATE
NAME

Marking Scheme

CLASS

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INDEX
NUMBER

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MATHEMATICS

Paper 1

4052/01

21 August 2025

2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

The number of marks is given in brackets [] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142.

For Examiner's Use

90

Mathematical Formulae

Compound interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

Answer **all** the questions.

- 1 (a) Given $a = \frac{v^2 - t^2}{2s}$, express v in terms of a , t and s .

$$\begin{aligned} a &= \frac{v^2 - t^2}{2s} \\ 2as &= v^2 - t^2 && \text{[transform to non-fractional equation]} \\ v^2 &= 2as + t^2 \\ v &= \pm\sqrt{2as + t^2} \end{aligned}$$

Answer $v =$ _____ [2]

- (b) Simplify $\frac{xy}{x^2 - 4xy + 4y^2} - \frac{2y}{4y - 2x}$.

$$\begin{aligned} &= \frac{xy}{(x-2y)^2} - \frac{y}{2y-x} \quad \text{(factorise into a square)} \\ &= \frac{xy}{(x-2y)^2} + \frac{y}{x-2y} \\ &= \frac{xy + xy - 2y^2}{(x-2y)^2} \quad \text{(Attempt to change to common denominator)} \\ &= \frac{2xy - 2y^2}{(x-2y)^2} \quad \left(\text{or } \frac{2y(x-y)}{(x-2y)^2} \text{ or } \frac{2xy - 2y^2}{x^2 - 4xy + 4y^2} \right) \end{aligned}$$

Answer _____ [3]

- 2 The diameter of a solid sphere is 7.12 cm, measured correct to 3 significant figures. Its density is 2.2 g/cm³. Find the smallest possible mass of the sphere.

$$\begin{aligned} &\text{Mass} \\ &= \frac{4}{3}\pi \left(\frac{7.115}{2} \right)^3 \times 2.2 \quad \text{(See 7.115)} \\ &= 415 \text{ g} \end{aligned}$$

Answer _____ g [2]

- 3 An area of 512 km^2 is represented by 50 cm^2 on a map.
Find the scale of the map in the form $1 : n$.

$$\begin{aligned} 512 \text{ km}^2 &: 50 \text{ cm}^2 \\ 256 \text{ km}^2 &: 25 \text{ cm}^2 \\ 16 \text{ km} &: 5 \text{ cm} \quad (\text{Square root}) \\ 3.2 \text{ km} &: 1 \text{ cm} \\ 320\,000 &: 1 \end{aligned}$$

Answer

$$1 : 320\,000$$

[2]

- 4 (a) Factorise completely $m^3 + 30 + 5m + 6m^2$.

$$\begin{aligned} &= m^3 + 6m^2 + 5m + 30 \\ &= m^2(m + 6) + 5(m + 6) \quad (2 \text{ groups of factorisation}) \\ &= (m + 6)(m^2 + 5) \end{aligned}$$

[2]

- (b) Given that m is a positive integer, explain why $m^3 + 30 + 5m + 6m^2$ is not a prime number.

Its factors $(m + 6)$ and $(m^2 + 5)$ are whole numbers more than 1.
Hence, it's not a prime number.

[1]

- 5 Mary cycles 2.8 km at an average speed of 12 km/h and takes a break.
She then continues to cycle a further 1.4 km in 5 minutes .

Given that her average speed for the whole journey is $9\frac{1}{3} \text{ km/h}$, find the duration of her break.

$$\text{Time taken for first part} = \frac{2.8}{12} = \frac{7}{30} \text{ h}$$

Let $x \text{ h}$ be her break time.

$$9\frac{1}{3} = \frac{2.8 + 1.4}{\frac{7}{30} + x + \frac{1}{12}} \quad \text{-- M1}$$

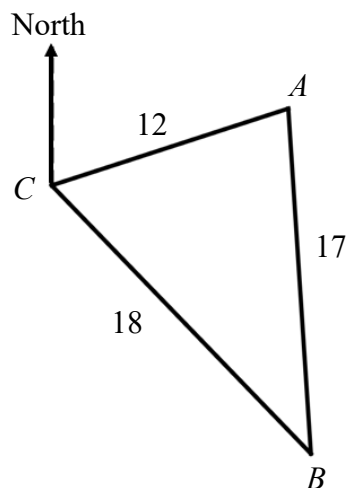
$$\frac{19}{60} + x = \frac{4.2}{9\frac{1}{3}}$$

$$x = \frac{2}{15} \quad (\text{or } 0.133 \text{ h})$$

h

[2]

6 (i)



The diagram below shows 3 lighthouses A , B and C at sea.

$AC = 12$ km. $BC = 18$ km. $AB = 17$ km.

The bearing of B from C is 136° .

Find the bearing of A from C .

$$\begin{aligned}\cos \angle ACB &= \frac{12^2 + 18^2 - 17^2}{2(12)(18)} \\ &= 0.41435 \text{ (5 s.f.)}\end{aligned}$$

$$\angle ACB = 65.522^\circ \text{ (3 dp)}$$

$$\begin{aligned}\text{Bearing of } A \text{ from } C &= 136^\circ - 65.522^\circ \\ &= 070.5^\circ \text{ (1 dp) (with zero in front)}\end{aligned}$$

° [2]

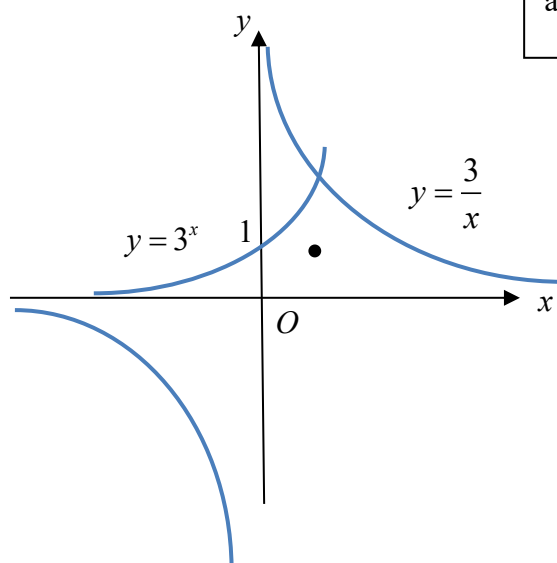
(ii) Find the area of the region enclosed by triangle ABC .

$$\begin{aligned}\text{Area of } ABC &= \frac{1}{2}(12)(18)\sin 65.522^\circ \\ &= 98.3 \text{ km}^2\end{aligned}$$

Answerkm² [2]

- 7 On the axes given, sketch the graphs $y = 3^x$ and $y = \frac{3}{x}$, indicating the x - and y -intercepts where relevant.
The point $(1, 1)$ is marked in the diagram.

Answer



[2]

- 8 (a) $\varepsilon = \{\text{all real numbers}\}$
 $A = \{\text{all perfect cubes}\}$
 $B = \{\text{all rational numbers}\}$
 $C = \{\text{all recurring decimals}\}$
 $D = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

- (i) List all the elements contained in the set $A \cap D$.

Answer

1, 8

[1]

- (ii) Explain why $C \cap B' = \phi$.

Answer

B' are irrational numbers which are **non-recurring** decimal numbers.

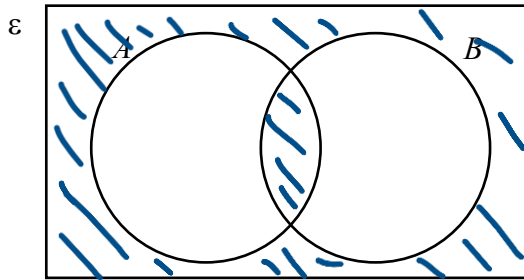
Therefore, $C \cap B' = \phi$.

(Or saying rational numbers are recurring decimals)

]

- (b) On the Venn diagram, shade the region(s) which represent $(A' \cap B') \cup (A \cap B)$.

Answer



[1]

- 9 A quadratic curve $y = -x^2 + mx + 3.5$ has a turning point at $(-1.5, p)$, where both m and p are constants. Find the value of m and of p .

Completed square form:

$$y = -(x + \frac{3}{2})^2 + p \quad \text{(Completed square form)}$$

$$= -(x^2 + 3x + \frac{9}{4}) + p \quad [\text{or see } y = -(x - \frac{m}{2})^2 + \frac{m^2}{4} + 3.5]$$

$$= -x^2 - 3x - \frac{9}{4} + p$$

Comparing coefficients with $y = -x^2 + mx + 3.5$: (Comparing)

$$m = -3;$$

$$-\frac{9}{4} + p = 3.5$$

$$\rightarrow p = \frac{23}{4}$$

Alternative method

Completed square form:

$$y = -(x - \frac{m}{2})^2 + \frac{m^2}{4} + 3.5$$

Comparing coefficients with $y = -(x - \frac{m}{2})^2 + \frac{m^2}{4} + 3.5$: (Comparing)

$$\frac{m}{2} = -1.5 \rightarrow m = -3;$$

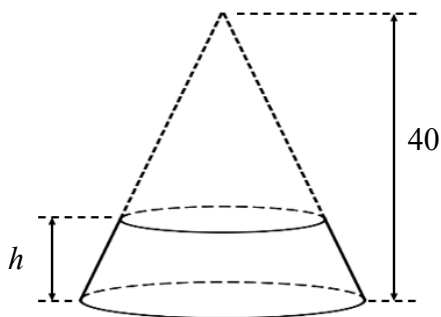
$$\frac{m^2}{4} + 3.5 = p$$

$$\rightarrow p = \frac{23}{4}$$

- 10** A pet dish bowl is in the shape of a frustum, as shown in the diagram. It is formed by removing a smaller cone from the top of a larger cone.

The smaller cone is similar to the original cone, and its volume is $\frac{3}{8}$ of the volume of the original cone. The original cone has a height of 40 cm.

- (a)** Find h , the height of the frustum.



$$\sqrt[3]{\frac{3}{8}} = \frac{40-h}{40} \text{--[using cube root]}$$

$$h = 40 - 40 \times \sqrt[3]{\frac{3}{8}}$$

$$h = 11.2 \text{ (3s.f.)}$$

Answer $h =$ _____ [2]

- (b)** Given that the curved surface area of the frustum is 966 cm^2 , find the curved surface area of the smaller cone.

$$\frac{A_{\text{smaller cone}}}{A_{\text{larger cone}}} = \left[\left(\frac{3}{8} \right)^{\frac{1}{3}} \right]^2 \text{--[squaring the ratio of lengths]}$$

$$\frac{A_{\text{smaller cone}}}{A_{\text{smaller cone}} + 966} = \left(\frac{3}{8} \right)^{\frac{2}{3}}$$

$$A_{\text{smaller cone}} = \left(\frac{3}{8} \right)^{\frac{2}{3}} (A_{\text{smaller cone}} + 966)$$

$$A_{\text{smaller cone}} = \frac{\left(\frac{3}{8} \right)^{\frac{2}{3}} (966)}{1 - \left(\frac{3}{8} \right)^{\frac{2}{3}}} = 1050 \text{ cm}^2 \text{ (3 s.f.)}$$

Answer _____ cm^2 [2]

- 11 The table shows the income tax rates for different tax brackets.

Chargeable income	Income tax rate (%)
First \$20 000	0
Next \$10 000	2
Next \$10 000	3.5
Next \$40 000	7
Next \$40 000	11.5

In 2022, given that David's chargeable income is \$60,500, find his income tax payable for YA 2022.

$$\begin{aligned}
 &\text{Income tax payable} \\
 &= \$10000 \times \frac{2}{100} + \$10000 \times \frac{3.5}{100} + \$20500 \times \frac{7}{100} \\
 &= \$1985
 \end{aligned}$$

Answer \$ _____ [2]

- 12 (a) Express 630 as a product of its prime factors.

$$630 = 2 \times 3^2 \times 5 \times 7$$

Answer _____ [1]

- (b) Find the smallest integer value of m such that $630m$ is a perfect square.

$$m = 2 \times 5 \times 7 = 70$$

Answer $m =$ _____ [1]

- (c) Given that $588 = 2^2 \times 3 \times 7^2$, find the smallest integer value of k such that $630k$ is a multiple of 588.

$$k = 2 \times 7 = 14$$

Answer $k =$ _____ [1]

- (d) A rectangular piece of paper measuring 588 mm and 630 mm is being cut into identical squares with no waste.
Find the minimum number of squares that can be obtained.

$$\begin{aligned}
 &\text{HCF} = 2 \times 3 \times 7 = 42 \text{ -- (find HCF)} \\
 &\text{No of pieces} = \frac{588 \times 630}{42^2} = 210
 \end{aligned}$$

Answer _____ [2]

13 Consider the pattern

$$L_1 : 4^2 - 1^2 = 15$$

$$L_2 : 5^2 - 2^2 = 21$$

$$L_3 : 6^2 - 3^2 = 27$$

$$L_4 : 7^2 - 4^2 = 33$$

⋮
⋮

$$L_n : \underline{\hspace{2cm}}$$

(a) Write down L_n of the pattern.

$$L_n = (n+3)^2 - n^2 = 6n+9$$

Answer $L_n = \underline{\hspace{2cm}}$ [1]

(b) Explain, with working, whether 3336 is a term of the sequence 15, 21, 27, 33.....

Answer

$$3336 = 6n + 9 \text{ -- [M1](Finding } n)$$

$$n = 554.5$$

No, as n is not a (positive) integer / whole number.

[2]

(c) Find the exact value of $L_1 + L_4 + L_7 + L_{10} + L_{13} + \dots L_{97} + L_{100}$.

$$L_1 + L_4 + L_7 + L_{10} + L_{13} + \dots L_{97} + L_{100}$$

$$= 4^2 - 1^2 + 7^2 - 4^2 + 10^2 - 7^2 + \dots 100^2 - 97^2 + 103^2 - 100^2$$

--(replacing the L terms with $a^2 - b^2$)

$$= 103^2 - 1^2$$

$$= 10608 \text{ -- [A1]}$$

Answer $\underline{\hspace{2cm}}$ [2]

- 14 (a) A polygon has three of its interior angles, measuring 150° , 160° and 170° .
The other interior angles are 130° each.
Find n , the number of sides of the polygon.

$$\begin{aligned} 150^\circ + 160^\circ + 170^\circ + (n-3)(130^\circ) &= (n-2)(180^\circ) \\ 480^\circ + 130n - 390^\circ &= 180^\circ n - 360^\circ \\ 450^\circ &= 50n \\ n &= 9 \end{aligned}$$

Answer $n =$ _____ [2]

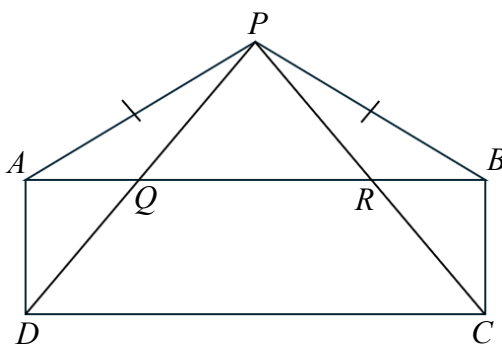
- (b) Explain why it is not possible for a regular polygon to have an interior angle of 145° .

Answer

Its exterior angle is 35° . $n = \frac{360}{35} = 10\frac{2}{7}$ -- (finding n)
Since the number of sides is not a whole number / (positive) integer, it is not possible to be a regular polygon to have an interior angle of 145° .

[2]

- 15 The diagram shows a rectangle $ABCD$ and an isosceles triangle PAB in which $PA = PB$.
 PD and PC meet AB at Q and R respectively.
Prove that triangle PDC is an isosceles triangle.



Answer

$PA = PB$ (given) (S)
 $AD = BC$ (opp. sides of rectangle) (S)
 $\angle PAB = \angle PBA$ (base angles of isosceles triangle)
 $\angle PAD = \angle PAB + 90^\circ$
 $\quad = \angle PBA + 90^\circ$ (A) (**finding** $\angle PAD = \angle PBC$)
 $\quad = \angle PBC$
Therefore, Triangle $PAD \cong$ Triangle PBC (SAS)
 $\therefore PD = PC$,
triangle is an isosceles triangle.

[3]

- 16 The table below shows the number of books students read in a certain week.

Number of books	0	1	2	3	4	5
Number of students	9	13	16	21	x	17

- (a) If the mode is 3, state the largest possible value of x .

Answer $x =$ 20 [1]

- (b) If the mean is 3, find the value of x .

$$\begin{aligned} \frac{0+13+32+63+4x+85}{9+13+16+21+x+17} &= 3 \\ \frac{193+4x}{76+x} &= 3 \\ 193+4x &= 228+3x \\ x &= 35 \end{aligned}$$

Answer $x =$ _____ [2]

- (c) Melanie claims that it is not possible for the median to be 2.
Explain whether you agree with Melanie's claim.

To find the smallest possible median is when $x = 0$,
(the smallest possible value of x)

– **(attempt to find smallest possible median)**

Alternative solution:

Stating that Left side has $9+13+16 = 38$ students;

Right side has $21 + 17 = 38$ students.

Hence the smallest median is 2.5

$$\text{The smallest possible median} = \frac{2+3}{2} = 2.5$$

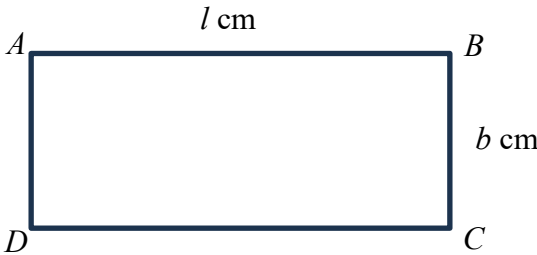
At The smallest possible median is 2.5, and it will not be 2. Melanie is correct. -- (also accept finding median to be more than 2)

[2]

- 17 The diagram shows a sheet of rectangular paper $ABCD$.
 The length is l cm. The breadth is b cm.
 This sheet can be rolled to form an open cylinder in two different ways:

Method 1: By joining the breadths together.

Method 2: By joining the lengths together.



Justify with mathematical workings, why the volume of the open cylinder, formed using **Method 1** is larger than that formed using **Method 2**, for all real values of l and b .

Answer

	Method 1	Method 2	
Finding radius	$2\pi r_1 = l$ $r_1 = \frac{l}{2\pi}$	$2\pi r_2 = b$ $r_2 = \frac{b}{2\pi}$	(Correct radii for both)
Volume	$= \pi \left(\frac{l}{2\pi} \right)^2 \times b$ $= \frac{bl^2}{4\pi}$	$= \pi \left(\frac{b}{2\pi} \right)^2 \times l$ $= \frac{b^2l}{4\pi}$	(Finding volume)

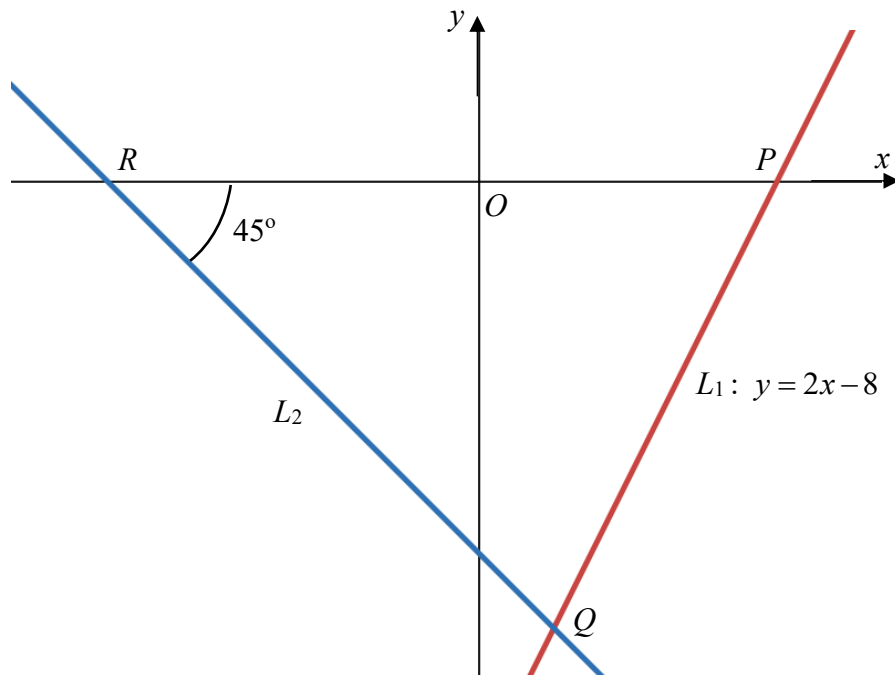
Since $l > b \rightarrow \frac{bl^2}{4\pi} > \frac{b^2l}{4\pi}$, -- (follow through from above)

[Alternatively, $\left(\frac{bl}{4\pi} \right) l > \left(\frac{bl}{4\pi} \right) b$]

the volume from method 1 is larger.

[4]

- 18** In the figure below, line L_1 and line L_2 cut the x -axis at P and R respectively.
 The equation of L_1 is $y = 2x - 8$.
 Angle PRQ is 45° . PR is 9 units.
 Line L_2 meets line L_1 at Q .



- (a)** By finding the coordinates of R , show that the equation of Line L_2 is $y = -x - 5$.

Answer

$$P = (4, 0)$$

$$R = (-5, 0)$$

Gradient $= -\tan 45^\circ = -1$ --(or other similar methods to find gradient)

Therefore, $y = -x + c$

At $(-5, 0)$,

$$0 = 5 + c$$

$$c = -5$$

--(connect gradient and a point to find c)

Therefore, equation of $L_2 : y = -x - 5$ (Shown)

(b) Find the coordinates of Q .

$$y = 2x - 8 \quad \text{--(1)}$$

$$y = -x - 5 \quad \text{--(2)}$$

$$(1) = (2)$$

$$2x - 8 = -x - 5$$

$$x = 1 \rightarrow y = -6$$

$$Q = (1, -6)$$

(..... ,) [2]

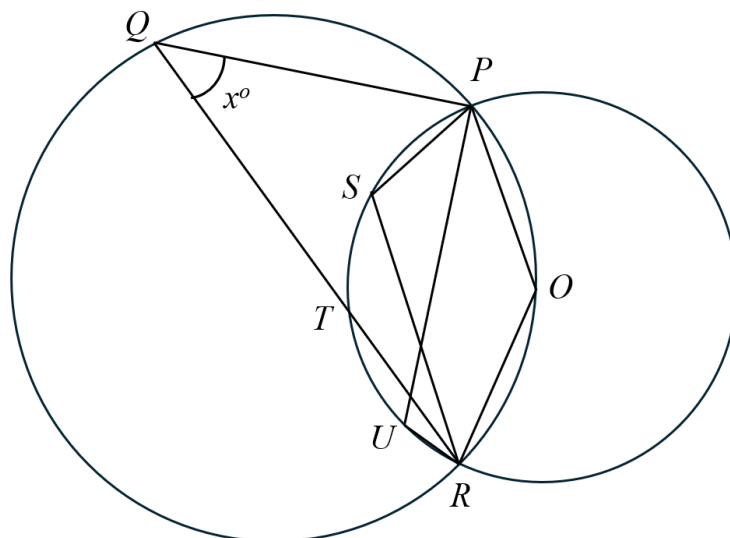
(c) Find the shortest distance from R to line PQ .

$$\begin{aligned} \text{Area of triangle} &= \frac{1}{2} \times 6 \times 9 \\ &= 27 \text{ units}^2 \end{aligned}$$

$$\begin{aligned} PQ &= \sqrt{(4-1)^2 + (0+6)^2} \quad \text{--(finding base of triangle)} \\ &= \sqrt{45} \text{ units} \end{aligned}$$

$$\text{Shortest distance} = \frac{27 \times 2}{\sqrt{45}} = 8.05 \text{ units (3s.f.)}$$

Answer units [3]



In the diagram, O is the centre of the smaller circle passing through the points P, S, T, U and R .

QTR is a straight line.

The points P, Q, R and O lie on the larger circle. It is given that angle $PQR = x^\circ$.

- (a) Find angle PSR , in terms of x .
Give a reason for each step of your working

Answer

$\angle POR = (180 - x)^\circ$ (angles in opp segments)
reflex $\angle POR = 360^\circ - (180 - x)^\circ$ (angles at a pt)
$= (180 + x)^\circ$
$\angle PSR = \frac{(180 + x)^\circ}{2}$ (angle at centre = 2 x angle at circumference)
(also accept $\left(90 + \frac{x}{2}\right)$)

[3]

- (b) It is now given that angle PUR is 105° , find the value of x .

$\angle PUR = \frac{(180 + x)^\circ}{2} \text{ (angles in the same segment)}$ $105 = \frac{(180 + x)}{2}$ $x = 30$
--

Answer $x =$ _____ [2]

- 20 There are 3 statues located at points A , B and C respectively.
The diagram in the space below shows the positions of points A and B .

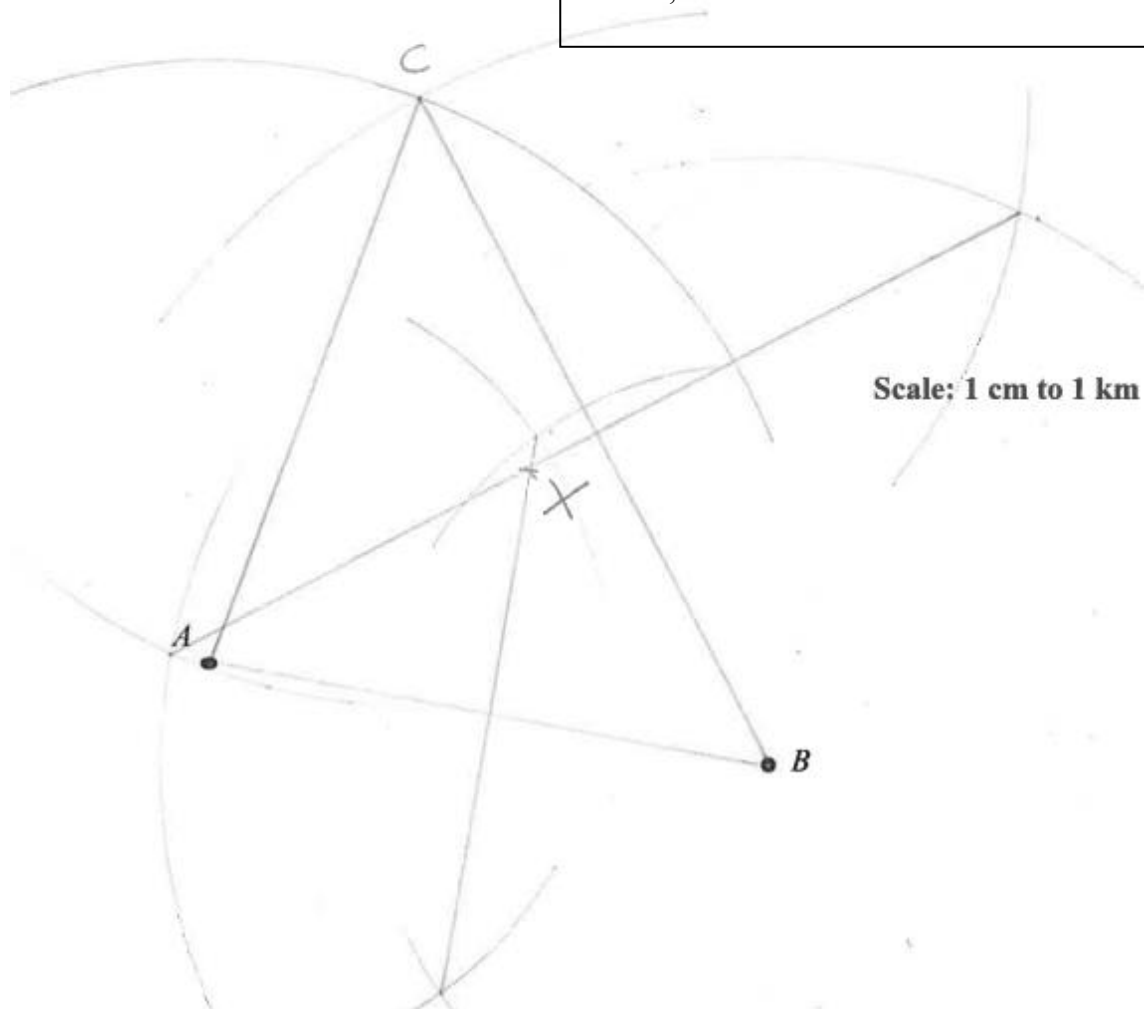
- (a) It is given that AC is 8 km and BC is 10 km.
Using appropriate constructions, find and label the position of point C .

[1]

Answer

Draw 2 arcs

Find C , the intersection of the 2 arcs



- (b) There is a treasure chest buried in a location that is equidistant from A , B and C . Find and label this point X in the diagram above.

[2]

Draw at least 2 perpendicular bisectors

Find X , the intersection of the 2 perpendicular bisectors

- 21 P is inversely proportional to the square of Q .
Given that Q increases by 10%, find the percentage decrease in P .

$$\begin{aligned}
 P &= \frac{k}{Q^2}, \quad Q_{\text{new}} = 1.1Q ; \\
 P_{\text{new}} &= \frac{k}{(1.1Q)^2} \\
 &= \frac{k}{1.21Q^2} \\
 \text{Percentage change} &= \frac{\frac{k}{1.21Q^2} - \frac{k}{Q^2}}{\frac{k}{Q^2}} \times 100\% \quad (\text{finding \% change}) \\
 &= \left(\frac{1}{1.21} - 1 \right) \times 100\% = -17.4\% \quad (3\text{s.f.}) \\
 \text{Decrease \%} &= 17.4\%
 \end{aligned}$$

% [3]

- 22 In Australia, a 16-oz bottle of shampoo costs A\$60.
The shampoo company wants to price a 500ml bottle of the same shampoo at the same rate in France.

$$\text{€ } 1 = \text{A\$}1.65.$$

$$1 \text{ oz} = 29.5735 \text{ ml}.$$

What should the price be in euros (€) ?

Leave your answer to the nearest 10 cents.

$$\text{In Australia, a 16-oz bottle costs A\$60} = \text{€} \frac{60}{1.65} \quad \text{--(Converting to euros)}$$

$$29.5735\text{ml (also 1 oz) of shampoo costs} = \text{€} \frac{60}{1.65} \div 16$$

$$\begin{aligned}
 500 \text{ ml of shampoo costs} &= \text{€} \frac{60}{1.65} \div 16 \times \frac{500}{29.5735} \quad \text{--(Converting to ml)} \\
 &= \text{€}38.40 \quad (\text{to nearest 10 cents})
 \end{aligned}$$

Answer € _____ [3]

- 23** A bag contains 12 counters, n of which are green and the remainder are blue. One counter is drawn at random and not replaced.

(a) Write down, in terms of n , the probability that the counter is blue.

Answer

$\frac{12-n}{12}$

 [1]

A second counter is then drawn at random.

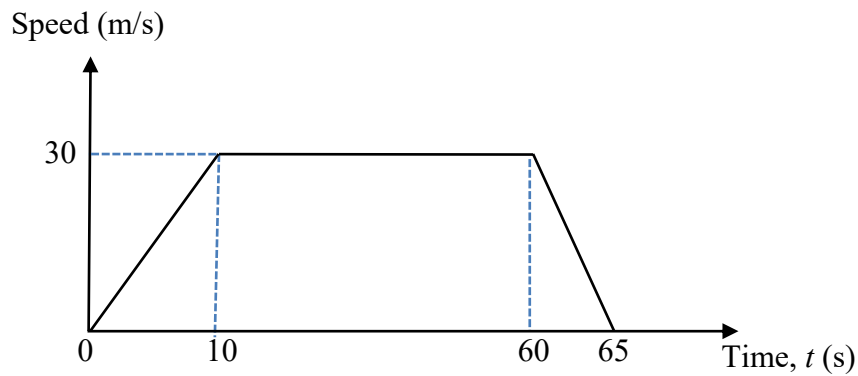
- (b)** Given that the probability that both counters drawn are blue is $\frac{5}{22}$, form an equation in the form $n^2 + bn + c = 0$, where b and c are integers.

Answer

$\frac{12-n}{12} \times \frac{11-n}{11} = \frac{(12-n)(11-n)}{132}$ $\frac{(12-n)(11-n)}{132} = \frac{5}{22}$ $(12-n)(11-n) = 30$ $n^2 - 23n + 102 = 0$

[3]

24 The diagram shows the speed-time graph of a car as it travelled from point *A* to *B*.



- (a) Find the deceleration of the car from 60s to 65s.

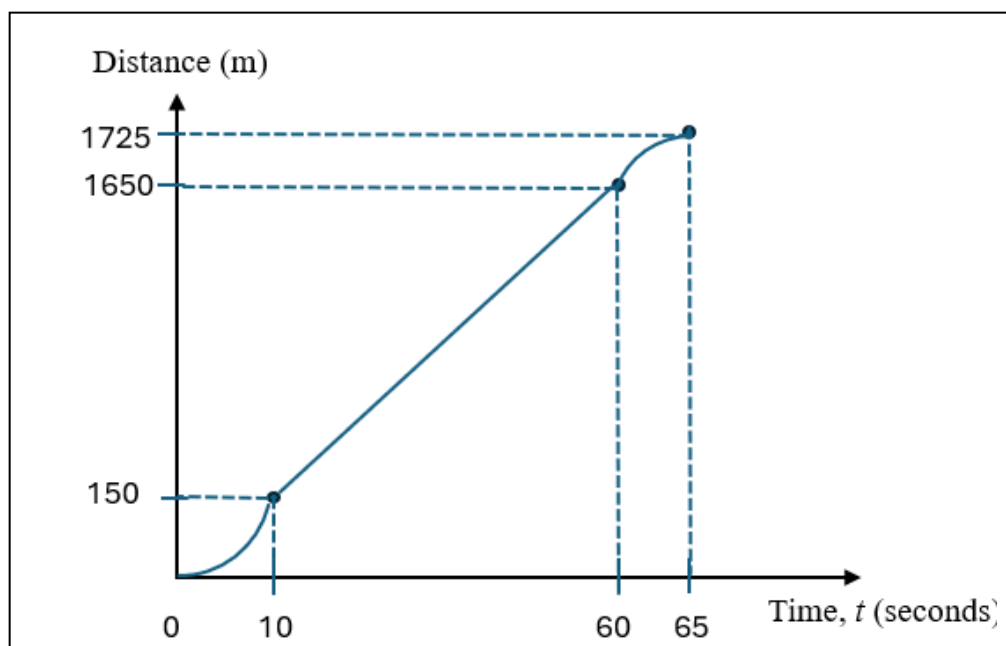
$$\frac{30}{5} = 6 \text{ m/s}^2$$

Answerm/s² [1]

The area beneath the speed-time graph represents the distance travelled by the object.

- (b) Draw the distance-time graph to represent the journey.

Answer



[3]

End of Paper

PRELIMINARY EXAMINATION 2025

SECONDARY FOUR

CANDIDATE
NAME

MARK SCHEME

CLASS

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INDEX
NUMBER

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MATHEMATICS

Paper 2

4052/02

22 August 2025

2 hours 15 minutes

Candidates answer on the Question Paper.

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For Examiner's Use

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Mathematical Formulae

Compound interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

Answer **all** the questions.

1 (a) Solve the inequality $1 + 4y \leq \frac{2y-1}{2} < 0$.

$$2 + 8y \leq 2y - 1 < 0$$

$$2 + 8y \leq 2y - 1 \quad \text{and} \quad 2y - 1 < 0$$

$$y \leq -\frac{1}{2} \quad y < \frac{1}{2}$$

$$\therefore y \leq -\frac{1}{2}$$

Answer _____ [2]

(b) Solve $-(1-x)\left(\frac{4}{5}x-1\right) = 3$.

$$(x-1)\left(\frac{4}{5}x-1\right) = 3$$

$$\frac{4}{5}x^2 - \frac{9}{5}x + 1 = 3$$

$$4x^2 - 9x - 10 = 0$$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(4)(-10)}}{2(4)}$$

$$x = -0.816 \text{ or } x = 3.07 \text{ (3 s.f.)}$$

Answer $x = \dots\dots\dots$ and $x = \dots\dots\dots$ [3]

- (c) (i) Given that $243^p \times 2^{5p} = 6^{6-p}$, find the value of p .

$$3^{5p} \times 2^{5p} = 6^{6-p}$$

$$6^{5p} = 6^{6-p}$$

$$5p = 6 - p$$

$$p = 1$$

Answer $p =$ _____ [2]

- (ii) Simplify $\left(\frac{16p^4}{q^8}\right)^{-\frac{3}{2}} \div pq^5$ and leave your answer in positive index notation.

$$= \left(\frac{q^8}{16p^4}\right)^{\frac{3}{2}} \div pq^5$$

$$= \frac{q^{12}}{64p^6} \times \frac{1}{pq^5}$$

$$= \frac{q^7}{64p^7}$$

Answer _____ [3]

- 2 (a) The diameter of a red blood cell is 8.2×10^{-6} m.
Express this value as a decimal, in millimetres.

$$\begin{aligned} &8.2 \times 10^{-3} \text{ mm} \\ &= 0.0082 \text{ mm} \end{aligned}$$

Answer _____ mm [1]

- (b) In 2024, Changi Airport handled 67.7 million passengers.
Calculate the mean number of passengers handled by the airport each month.
Leave your answer in standard form, correct to 2 significant figures.

$$\begin{aligned} &\frac{67.7 \times 10^6}{12} \\ &= 5.6 \times 10^6 \end{aligned}$$

Answer _____ [2]

- (c) LBB Bank currently charges 3% per year compound interest on all loans, compounded once every three months.
The bank manager proposes to adjust the frequency of interest compounded to once every month instead, so as to increase the earnings of the bank.

Do you agree with the bank manager's proposal?
Explain your answer with clear workings.

Answer

Compounded every 3 months:

$$P \left(1 + \frac{3\%}{4} \right)^{4n} = P(1.0075)^{4n}$$

Compounded every month:

$$\begin{aligned} P \left(1 + \frac{3\%}{12} \right)^{12n} &= P(1.0025)^{12n} \\ &= P(1.007518766)^{4n} \end{aligned}$$

I agree with the bank manager's proposal, as the total earnings over the same period of
time is more when the frequency of interest compounded is once every month.

[3]

- 3 A bakery selling assorted tarts operates for 7 days a week.
The matrix, $\mathbf{T}$, shows the number of tarts of different types that are made each day.

$$\mathbf{T} = \begin{pmatrix} \text{Apple} & \text{Banana} & \text{Cheese} \\ 55 & 35 & 70 \\ 40 & 30 & 30 \end{pmatrix} \begin{matrix} \text{Small} \\ \text{Large} \end{matrix}$$

The cost of making a small tart is \$0.80.
The cost of making a large tart is \$1.75.

- (a) Represent these costs in a 1×2 row matrix $\mathbf{N}$.

Answer $\mathbf{N} = (0.8 \quad 1.75)$ [1]

- (b) Evaluate the matrix $\mathbf{C} = 7\mathbf{NT}$.

$$\begin{aligned} \mathbf{C} &= 7(0.8 \quad 1.75) \begin{pmatrix} 55 & 35 & 70 \\ 40 & 30 & 30 \end{pmatrix} \\ &= 7(114 \quad 80.5 \quad 108.5) \\ &= (798 \quad 563.5 \quad 759.5) \end{aligned}$$

Answer $\mathbf{C} =$ [2]

- (c) State what each of the elements of $\mathbf{C}$ represent.
It represents the total cost of making apple, banana and cheese tarts respectively,

in a week.

[1]

- (d) The tarts are sold at 55% more than the cost to make them.
On a particular day, the number of banana tarts to cheese tarts sold is in the ratio 5 : 7 while the number of apple tarts to banana tarts sold is in the ratio 5 : 4. A total of 41 unsold tarts were given away.

If the number of small cheese tarts sold is twice the number of large cheese tarts sold, calculate the total amount earned by the bakery from selling cheese tarts on that day.

Apple : Banana : Cheese

$$\begin{array}{l} 5 : 4 \\ 5 : 7 \\ \therefore 25 : 20 : 28 \end{array}$$

$$\begin{aligned} \text{No. of cheese tarts sold} &= \frac{260 - 41}{25 + 20 + 28} \times 28 \\ &= \frac{219}{73} \times 28 = 84 \end{aligned}$$

Total amount earned from selling cheese tarts for the day

$$\begin{aligned} &= \left(\frac{2}{3} \times 84 \times 1.55 \times 0.80 \right) + \left(\frac{1}{3} \times 84 \times 1.55 \times 1.75 \right) \\ &= \$145.39 \end{aligned}$$

Answer \$ _____ [3]

- 4 (a) The variables x and y are connected by the equation $y = \frac{7}{x} + 2x - 11$.

Some corresponding values are given in the table below.

Values are given to 1 decimal place where appropriate.

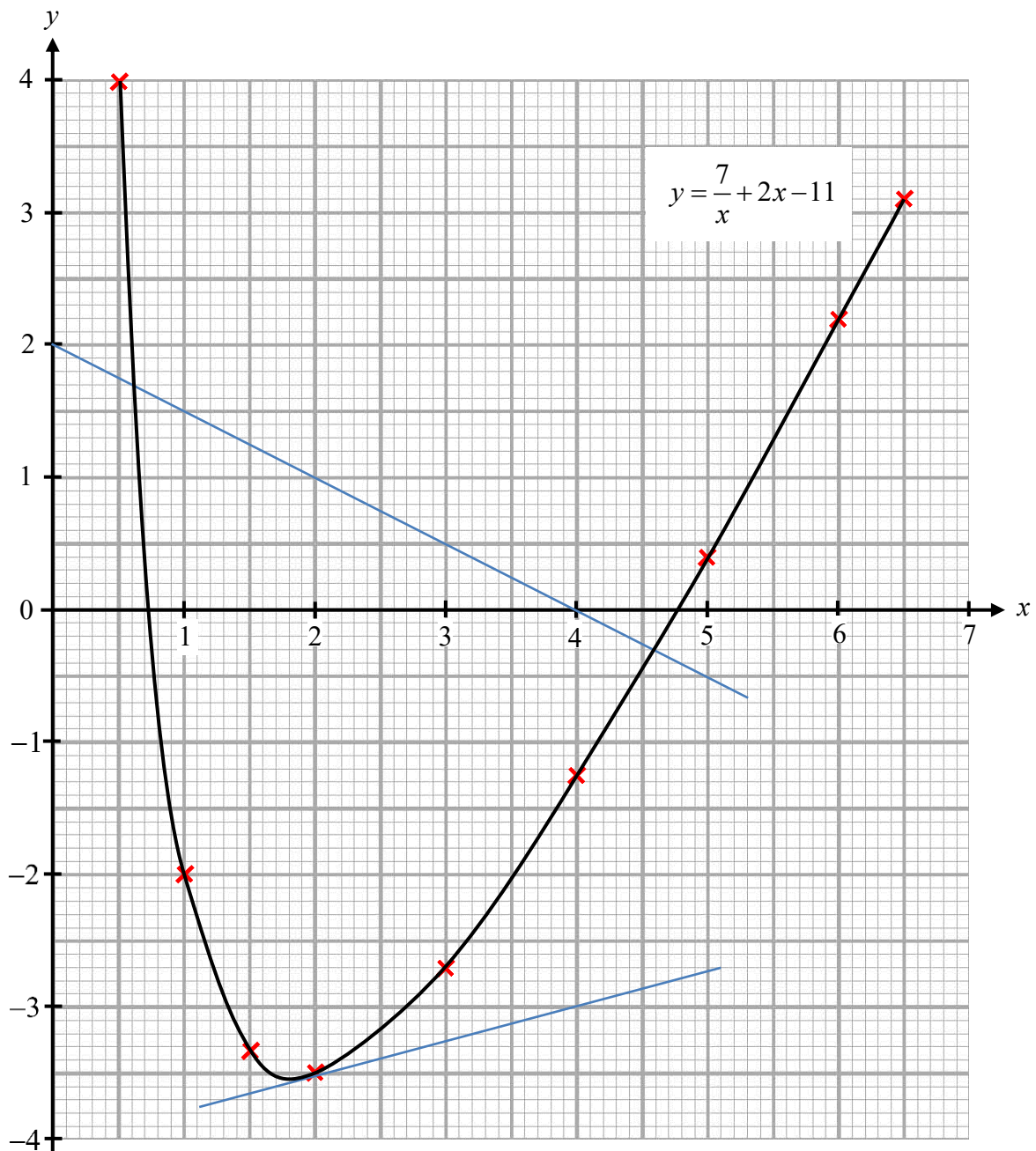
x	0.5	1	1.5	2	3	4	5	6	6.5
y	4	-2	-3.3	-3.5	-2.7	-1.25	0.4	2.2	3.1

Fill in the missing value in the table.

[1]

- (b) On the grid below, draw the graph of $y = \frac{7}{x} + 2x - 11$ for $0.5 \leq x \leq 6.5$.

[2]



- (c) By drawing a tangent, find the gradient of the curve at $x = 2$.

$$\begin{aligned}\text{Gradient} &= \frac{-3 - (-3.5)}{4 - 2} \\ &= 0.25 \text{ } (\pm 0.13)\end{aligned}$$

Answer Gradient = _____ [2]

- (d) Using your graph,

- (i) explain why the equation $2x + \frac{7}{x} - 7 = 0$ has no solution.

Answer

$$2x + \frac{7}{x} - 11 = -4$$

The line $y = -4$ is below the minimum point of/lies entirely below the curve

$y = 2x + \frac{7}{x} - 11$ and therefore, will not intersect the curve at any point.

[2]

- (ii) find the x -coordinate of the points where the line $2y = 4 - x$ intersects the curve.

Draw $y = -\frac{1}{2}x + 2$

From graph, x -coordinate of points = 0.6 and 4.6 (± 0.1)

Answer $x = \dots\dots\dots$ and $x = \dots\dots\dots$ [2]

- (iii) These values of x are the solutions of the equation $Ax^2 + Bx + 14 = 0$.

Find the value of A and of B .

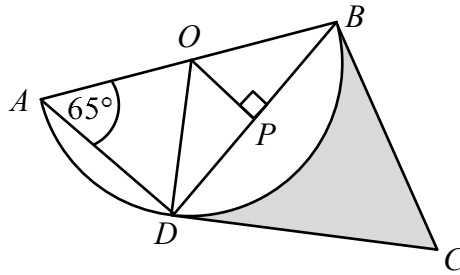
$$y = -\frac{1}{2}x + 2 \text{ ---(1)} \qquad y = 2x + \frac{7}{x} - 11 \text{ ---(2)}$$

$$\begin{aligned}\text{Sub (1) into (2):} \quad & -\frac{1}{2}x + 2 = 2x + \frac{7}{x} - 11 \\ & -\frac{1}{2}x + 2 = 2x + \frac{7}{x} - 11 \\ & -x^2 + 4x = 4x^2 + 14 - 22x \\ & 0 = 5x^2 - 26x + 14\end{aligned}$$

By comparing coefficients, $A = 5$ and $B = -26$

Answer $A = \dots\dots\dots$, $B = \dots\dots\dots$ [3]

- 5 The figure shows a semi-circle $AOBD$ with centre O and OP is perpendicular to BD . BC and CD are tangents to the semi-circle at B and D .



- (a) Show that triangles ABD and OBP are similar.
Give a reason for each statement you make.

Answer

$$\left. \begin{array}{l} \angle ADB = 90^\circ \text{ (angle in semicircle)} \\ \text{So, } \angle ADB = \angle OPB \\ \angle ABD \text{ is a common angle.} \end{array} \right\}$$

$\therefore \triangle ABD$ is similar to $\triangle OBP$. (Angle-angle similarity)

[2]

- (b) Find the ratio $\frac{\text{area of triangle } OBP}{\text{area of triangle } AOD}$.

$$\left. \begin{array}{l} \text{Since } \frac{\text{area of triangle } OBP}{\text{area of triangle } OBD} = \frac{1}{2} \\ \text{and } \text{area of triangle } OBD = \text{area of triangle } AOD, \end{array} \right\}$$

$$\frac{\text{area of triangle } OBP}{\text{area of triangle } AOD} = \frac{1}{2}$$

Answer _____ [2]

- (c) Show that angle $BOD = 2.2689$ radians, correct to 5 significant figures.

Answer

$$\begin{aligned} \angle BOP &= 65^\circ \text{ (similar triangles)} \\ \angle BOD &= (65^\circ \times 2) \times \frac{\pi}{180^\circ} \\ &= 2.268928 \text{ (7 s.f.)} \\ &= 2.2689 \text{ radians (5 s.f.) (shown)} \end{aligned}$$

[2]

- (d) It is given that the radius of the semi-circle is 4 cm and $BC = 8.58$ cm.

Calculate

- (i) the perimeter of the shaded part,

$$BC = DC = 8.58 \text{ cm (tangents from ext. pt.)}$$

$$\begin{aligned}\text{Perimeter} &= (8.58 \times 2) + 4(2.268928) \\ &= 26.2357 \text{ (5 s.f.)} \\ &= 26.2 \text{ cm (3 s.f.)}\end{aligned}$$

Answer _____ cm [2]

- (ii) the area of the shaded part.

$$\angle OBC = 90^\circ \text{ (tangent } \perp \text{ radius)}$$

$$\begin{aligned}\text{Area} &= \left(2 \times \frac{1}{2} \times 4 \times 8.58 \right) - \frac{1}{2} (4^2) (2.268928) \\ &= 16.169 \text{ (5 s.f.)} \\ &= 16.2 \text{ cm}^2 \text{ (3 s.f.)}\end{aligned}$$

Answer _____ cm^2 [3]

- 6 (a) (i) The position vector of point A and point B are $\begin{pmatrix} k \\ 6 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -2 \end{pmatrix}$ respectively.
Given that $|\overrightarrow{AB}| = 10$, find the negative value of k .

$$\overrightarrow{AB} = \begin{pmatrix} 2 \\ -2 \end{pmatrix} - \begin{pmatrix} k \\ 6 \end{pmatrix} = \begin{pmatrix} 2-k \\ -8 \end{pmatrix}$$

$$\begin{aligned} \text{So, } (2-k)^2 + (-8)^2 &= 10^2 \\ 4 - 4k + k^2 + 64 &= 100 \\ k^2 - 4k - 32 &= 0 \\ (k+4)(k-8) &= 0 \\ \therefore k &= -4, \quad 8 \text{ (rej)} \end{aligned}$$

Answer $k =$ _____ [3]

- (ii) C is the point $(5, -6)$.
Using the value of k found in part (a)(i), show that A , B and C are collinear.

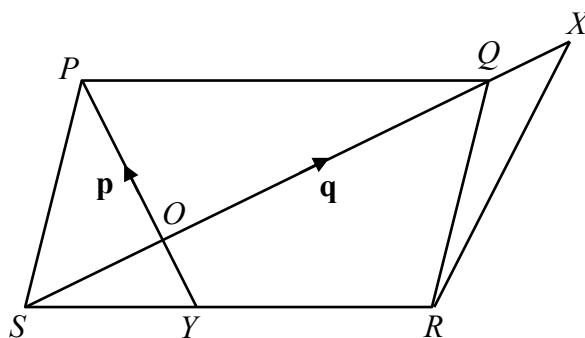
Answer

$$\overrightarrow{BC} = \begin{pmatrix} 5 \\ -6 \end{pmatrix} - \begin{pmatrix} 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$$

$$\overrightarrow{AB} = \begin{pmatrix} 6 \\ -8 \end{pmatrix} = 2 \begin{pmatrix} 3 \\ -4 \end{pmatrix} = 2\overrightarrow{BC}$$

Since $AB \parallel BC$ and B is a common point, A , B and C must be collinear.

- (b) The diagram shows a parallelogram $PQRS$ and a triangle QXR .
 QS is a diagonal of the parallelogram and Y is a point on RS such that $OS : QS = 3 : 10$.



It is given that $\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OQ} = \mathbf{q}$,

- (i) Express $\overrightarrow{QR}$, as simply as possible, in terms of $\mathbf{p}$ and $\mathbf{q}$,

$$\begin{aligned}\overrightarrow{SR} &= \overrightarrow{PQ} = -\mathbf{p} + \mathbf{q} \\ \overrightarrow{QR} &= \overrightarrow{QS} + \overrightarrow{SR} \\ &= -\frac{10}{7}\mathbf{q} + (-\mathbf{p} + \mathbf{q}) \\ &= -\mathbf{p} - \frac{3}{7}\mathbf{q}\end{aligned}$$

Answer $\overrightarrow{QR} = \underline{\hspace{2cm}}$ [3]

- (ii) If $\overrightarrow{QX} = m\overrightarrow{SQ}$ and $\overrightarrow{RX} = n\left(\frac{1}{5}\mathbf{p} + \frac{1}{7}\mathbf{q}\right)$, where m and n are rational numbers, find the value of m and of n .

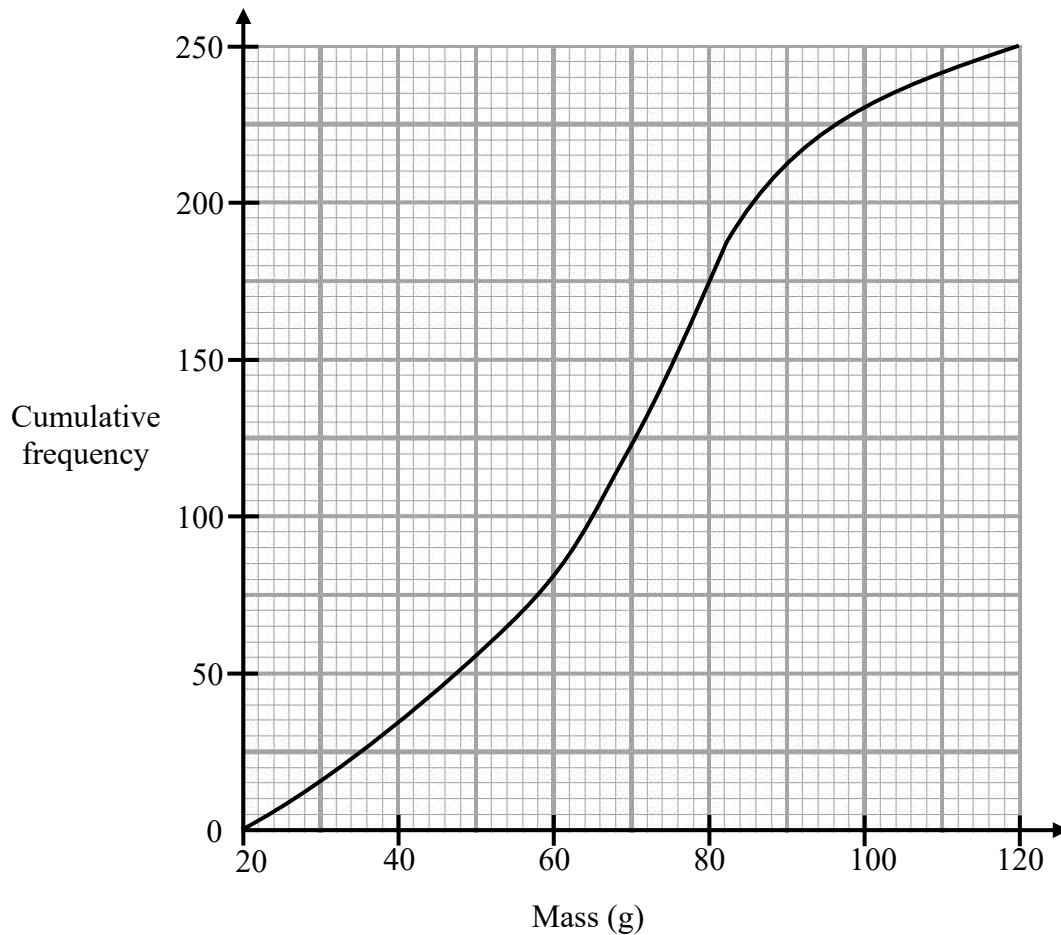
$$\begin{aligned}\overrightarrow{QX} &= \frac{10}{7}m\mathbf{q} \text{ --- (1)} \\ \overrightarrow{QX} &= \overrightarrow{QR} + \overrightarrow{RX} \\ &= -\mathbf{p} - \frac{3}{7}\mathbf{q} + n\left(\frac{1}{5}\mathbf{p} + \frac{1}{7}\mathbf{q}\right) \\ &= \left(\frac{1}{5}n - 1\right)\mathbf{p} + \left(\frac{1}{7}n - \frac{3}{7}\right)\mathbf{q} \text{ --- (2)}\end{aligned}$$

Comparing coefficients, $\frac{1}{5}n - 1 = 0 \quad \rightarrow n = 5$

$$\frac{1}{7}(5) - \frac{3}{7} = \frac{10}{7}m \quad \rightarrow m = \frac{1}{5}$$

Answer $m = \underline{\hspace{2cm}}, n = \underline{\hspace{2cm}}$ [4]

- 7 A farmer harvested 250 potatoes which were grown using Fertiliser X.
The cumulative frequency curve below shows the mass distribution of the potatoes.



- (a) The farmer only sells potatoes that weigh 56 g and above.

Using the curve, find the percentage of potatoes that can be sold.

$$\frac{250 - 70}{250} \times 100\% = 72\%$$

Answer _____ % [1]

- (b) (i) Complete the grouped frequency table for the mass of the potatoes.

Mass (m g)	$20 \leq m < 40$	$40 \leq m < 60$	$60 \leq m < 80$	$80 \leq m < 100$	$100 \leq m < 120$
Frequency	35	45	95	55	20

[1]

- (ii) Find the interval that contains the median mass.

Answer60.... $\leq m <$ 80.... [1]

(iii) Calculate an estimate of the mean mass.

Answer68.4 _____ g [1]

(iv) Calculate an estimate of the standard deviation of the masses.

Answer22.6 _____ g [1]

(c) Two of the potatoes are selected at random, without replacement.

Find the probability that

(i) none has a mass between 60 g and 100 g,

$$\begin{aligned} P(\text{none has mass between 60 g and 100 g}) &= \frac{100}{250} \times \frac{99}{249} \\ &= \frac{66}{415} \end{aligned}$$

Answer _____ [2]

(ii) one has a mass less than 60 g and the other has a mass greater than 100 g.

$$\begin{aligned} P(\text{one has mass less than 60 g and one has mass greater than 100 g}) \\ &= 2 \left(\frac{80}{250} \times \frac{20}{249} \right) \\ &= \frac{64}{1245} \end{aligned}$$

Answer _____ [2]

(d) The farmer harvested another 250 potatoes that were grown using Fertiliser Y.
The estimated mean mass of this batch of potatoes is 62 g.
The estimated standard deviation of the masses is 23 g.

Make two comments comparing the mass of the potatoes grown using Fertiliser X and those grown using Fertiliser Y.

Answer

On average, potatoes grown using Fertiliser X are heavier than those grown using

Fertiliser Y, as they have a higher mean mass of 68.4 g, compared to 62 g.

The spread of the mass is comparable for both batches of potatoes, with SD being

22.6 g and 23 g. / Mass of potatoes grown using Fertiliser X has a smaller spread, with

a lower SD of 22.6 g, rather than 23 g.

[2]

8 A factory produces bowls in two sizes, small size and large size.

- (a) It is found that x large bowls can be produced in a minute.
Write down an expression in terms of x , the time taken to produce 1 large bowl, **in seconds**.

$$\frac{60}{x}$$

Answer _____ s [1]

- (b) 4 more small bowls can be produced in a minute, compared to the large bowls.
Write down an expression in terms of x , the time taken to produce 1 small bowl, **in seconds**.

$$\frac{60}{x+4}$$

Answer _____ s [1]

- (c) Given that it takes 2.5 seconds longer to produce a large bowl than a small bowl, form an equation in x and show that it reduces to $x^2 + 4x - 96 = 0$.

Answer

$$\begin{aligned}\frac{60}{x} - \frac{60}{x+4} &= 2.5 \\ 60(x+4) - 60x &= 2.5x(x+4) \\ 60x + 240 - 60x &= 2.5x^2 + 10x \\ 2.5x^2 + 10x - 240 &= 0 \\ x^2 + 4x - 96 &= 0 \text{ (shown)}\end{aligned}$$

[2]

- (d) Solve the equation $x^2 + 4x - 96 = 0$.

$$\begin{aligned}x &= \frac{-4 \pm \sqrt{(-4)^2 - 4(1)(-96)}}{2(1)} \\ &= -12 \text{ or } 8\end{aligned}$$

Answer $x = \dots\dots\dots$ or $x = \dots\dots\dots$ [2]

- (e) Hence find the time taken to produce 3500 small bowls.
Give your answer in hours and minutes, correct to the nearest minute.

$$\begin{aligned}\text{Time taken} &= \frac{3500}{8+4} \\ &= 291\frac{6}{7} \text{ min} \\ &= 4 \text{ h } 52 \text{ min}\end{aligned}$$

Answer _____ h _____ min [2]

- (f) The factory sells each small bowl at \$0.35 and each large bowl at \$0.50.

Explain, with clear workings, why it is more profitable for the factory to produce only small bowls than only large bowls.

Answer

In t seconds,

$$\text{Amount earned from selling large bowls} = \$0.50 \times \frac{t}{60/8} = \$0.067t$$

$$\text{Amount earned from selling small bowls} = \$0.35 \times \frac{t}{60/12} = \$0.07t$$

The factory earns more from selling small bowls, given the same production time.

- 9 Ryan has set up a home theatre system in one of his rooms, which is in the shape of a cuboid with dimensions 4 m by 3.6 m by 3 m.

Figure 1 shows the three-dimensional layout of the room, where a television is mounted on the front wall $QRVU$ and two speakers are mounted on the back wall $PSWT$, at P and T respectively.

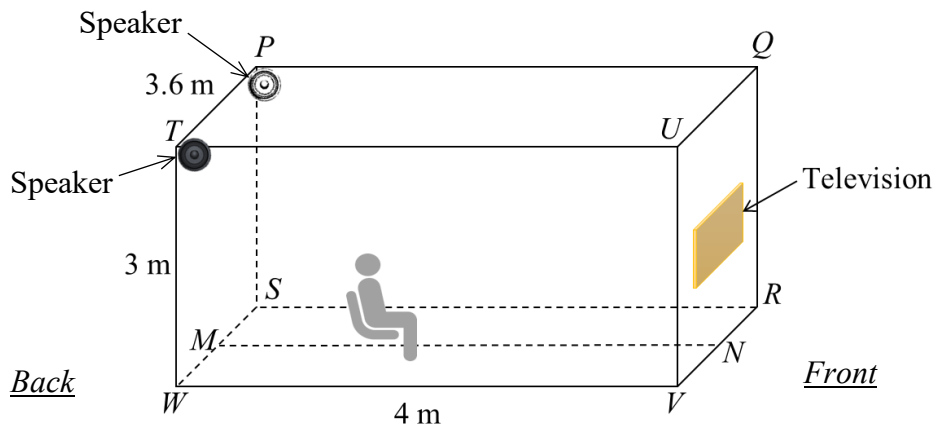


Figure 1

Ryan is planning to put an armchair along MN , which is a straight path that cuts across the centre of the room from the back to the front.

Figure 2 shows the cross-sectional view of him sitting in the armchair, where his eye level at X , is 1.24 m directly above the ground and d m away from the front wall. The centre of the television at Y , is 1.6 m directly above N .

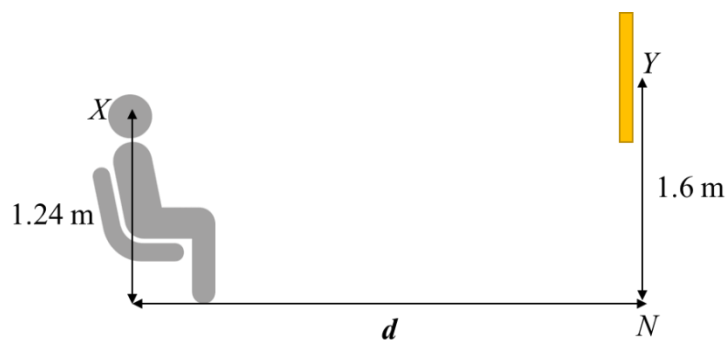


Figure 2

- (a) Find the angle of elevation from X to Y when $d = 1.2$ m.

Let the angle of elevation be θ .

$$\tan \theta = \frac{1.6 - 1.24}{1.2}$$

$$\theta = 16.699^\circ \text{ (5 s.f.)}$$

$$= 16.7^\circ \text{ (3 s.f.)}$$

Answer _____ $^\circ$ [2]

(b) Ryan found out that:

- To have an optimal view of the television, the angle of elevation from X to Y should be less than 15° .
- To have an optimal audio experience, the position of the two speakers and X should form an equilateral triangle.

Suggest a suitable value for d such that the position of the armchair will provide Ryan with best possible user experience.

Justify the decisions you make and show your calculations clearly.

Answer

1) When $\theta = 15^\circ$, $\tan 15^\circ = \frac{1.6 - 1.24}{d} \rightarrow d = 1.3435 \text{ m}$

To have optimal view of television:

For $\theta < 15^\circ$, $d > 1.3435 \text{ m}$

2) To have optimal audio experience:

$\triangle PXT$ needs to be equilateral $\rightarrow \angle TPX = \angle PXT = \angle XTP = 60^\circ$

Let perpendicular bisector of PT to X be m .

So, $\sin 60^\circ = \frac{m}{3.6} \rightarrow m = 3.1177 \text{ m}$

By Pythagoras' Theorem,

$$3.1177^2 = (4 - d)^2 + (3 - 1.24)^2$$

$$4 - d = \sqrt{3.1177^2 - 1.76^2}$$

$$d = 1.4266 \text{ m } (> 1.3435)$$

$\therefore$ Ryan should position the armchair at $d = 1.43 \text{ m}$ [DB1] because it will provide him with the best possible user experience, with **both** optimal view of the television and audio experience. [DB1]

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